

Using Interval Constrained Petri Nets for Regulation of Quality: The Case of Weight in Tobacco Factory

NABLI Lotfi and DHOUIBI Hédi

Abstract- The existence of maximal durations drastically modifies the performance evaluation in Discrete Event Systems (DES). The same particularity may be found on systems where the associated constraints do not concern the time. For example weight measures, in chemical industry, are used in order to control the quantity of consumed raw materials. This parameter also takes a fundamental part in the product quality as the correct transformation process is based upon a given percentage of each essence. Weight regulation therefore increases the global productivity of the system by decreasing the quantity of rejected products. The main contribution of this paper is to describe the behaviour of the Interval Constrained Petri Nets (ICPN). Their basic evolution rules are presented and the extension of the P-time PN structural analysis is discussed. An industrial application on a tobacco manufacturing plant, where the critical parameter is the weight is presented as an illustration.

Index terms: Petri Net, Manufacturing systems, Performance evaluation, Timed Petri Nets, Tolerant system.

1. INTRODUCTION

Manufacturing systems and particularly chemical and food industries are based on time constrained processes. More precisely, these processes are performed during certain time windows. Modelling such processes must specify, for every operation, either a minimal duration or an operating duration included in a time window, in order to ensure the product quality. Several studies describe appropriate models based on Timed Petri Nets and are able to model processes with time constraints [1, 2]. In the category of the workshops, the operations have temporal constraints that must be imperatively respected. The violation of these constraints can damage the health of consumers. The constraint violation causes a deterioration of quality or the stop of production.

In many processes, the deciding parameter for the quality and the cost is not time. However, this parameter must strictly belong to some validity intervals. The control must guarantee the fulfilment of these specifications. So a suitable model, rich in

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Nabli Lotfi and Dhouibi Hedi are with Unit of Automatic, Signal processing and Imaging (ATSI), National school Engineers of Monastir, Road of Kairouan, Monastir 5000, Tunisia.
(corresponding author, phone: 0021698503487; e-mail: lotfi.nabli@enim.rnu.tn)

analytical properties, is necessary to synthesize the needed controller.

The aim of this paper is to explain the use of Interval Constrained Petri Nets (ICPN) a sub-class of High Level Petri Nets with Abstract Marking (AM-HLPN) [3, 4]. ICPN allow one to model and guarantee a constraint on any parameter of a manufacturing process. They are applied to a tobacco factory unit where the speed of machines is constant. We ensure the product quality by verifying weight specifications. The production is optimised by minimizing losses in presence of variation.

First we introduce ICPN that are a functional abstraction of P-Time Petri Nets [1]. In this way, the extension of structural properties of P-Time PN can be justified under some conditions [5]. A theorem application is extended to our case study and will be illustrated in a practical manner.

2. INTERVAL CONSTRAINED PETRI NET

ICPN are introduced to extend the application field of P-Time PN by proceeding to a functional abstraction of the parameter associated places. Naturally, it is logical to find exactly the same mathematical definition of the tool. Nevertheless, if the restriction of the associated parameter to rational numbers is justified when this one is duration, it has no more justification in the case of modelling weight variation or position. Such parameters may take both negative and positive values.

Furthermore, the introduction of a new formalism is an opportunity to review the initial definition. Thus, we present in an unequivocal manner the marking as a multi-set. In the same way the transmission of a quantity conveyed by a token is represented explicitly.

2.1 Definition

An Interval Constrained Petri Net is an 8-uple $\langle R, M, IS, D, Val, Val_0, X, X_0 \rangle$ where:

- R is a marked Petri net [6],
- M is an application associating tokens to places as follow:

Let $m(p)$ be a place marking.

Let Va be a set of rational variables.

Let V be a non empty set of formulas using variables of V_a .

Let μV be a multiset defined on V .

$$m : P \rightarrow \mu V$$

$$p \in P \rightarrow m(p)$$

Let Val be an application: $M(P) \rightarrow \mu V$

$(k \in m(p), p) \rightarrow v \in \mu V$. Where k is a given token in p
 $k \rightarrow q_i \mid a_i \leq q_i \leq b_i$ where a_i and b_i are rational values fixed by IS

$M : P \rightarrow N$ (set of positive integer).

$$p \rightarrow \text{card}(m(p))$$

- $IS : P \rightarrow R \cup \{-\infty, +\infty\} \times R \cup \{-\infty, +\infty\}$ defines intervals associated to places. R is the set of real numbers.

$$p_i \rightarrow IS_i = [a_i, b_i] \text{ where } 0 \leq a_i \leq b_i$$

- D is an application that associates to each pair (place, marking) a rational variable q smaller or equal to b_i . This variable corresponds to the modification of the value associated value of a token in place p_i .

$$D: m(p) \times P \rightarrow V_a$$

$$\forall i, 1 \leq i \leq n, n = \text{Card}(P)$$

- X is an application that assigns to each variable a value.

$$X : V_a \rightarrow Q$$

$$va \rightarrow u \in Q$$

X sets q_i .

- X_0 defines the initial values of variables.
- Val associates to each token a formula of values in Q .

Val is an application of set of the tokens $m(P)$ in

$$V : m(P) \rightarrow V$$

- Val_0 defines the initial formulas associated to tokens.

A token in place p_i is taken into account in transition validations when it has reached a value between a_i and b_i . When the value is greater than b_i the token is said to be dead.

Of course, after the firing of the upstream transitions tokens are generated in its output places. Their associated values are equal to:

$$Val(k) + q_i(k).$$

The significations of q and $Val(k)$ are intentionally not fixed in order to provide a general model. Furthermore, a mathematical relationship between the

q in different places is supposed to exist. Nevertheless it is not mathematically defined.

As an example with P-time PN there is the following relation:

$$\forall q, dq/dt=1 \text{ where } t \text{ represents time.}$$

In ICPN the application X is not mathematically imposed. We will meet, for example, applications where q parameters represent weight variations of cigarettes. In this case, parameter values associated to pairs (place, token) are independent.

2.2 State definition

A state E is defined by a t-uple $\langle M, D, Val, X \rangle$ where:

- M , and Val are the above defined applications
- D and m assign a variable $q_i(k)$ to each token k in a place p_i .

A token k of the place p_i can take a part in the validation of output transitions if:

- $q_i(k) \in [a_i, b_i]$, where $[a_i, b_i]$ is the static interval associated to the place p_i . This token k dies when: $q_i(k) > b_i$
- X is an application which provides a value for each variable of V . Actually; X defines the real value of each q .

When X is not defined, there exists a way to make the model evolve. Furthermore, some mathematical properties may be outlined. It is the mathematical abstraction.

Definition 2.1: An abstraction on a set $\{x : A \bullet \psi\}$ may be interpreted as a set a value of the domain associated to the formula A . The A equation using x has to satisfy the constraint ψ .

The parameters defined with constrained variables will be specified, even though X is not defined, using the above definition:

$$A = Val_0(k) + \sum q_j,$$

The j index correspond to the visiting order of places by a given token

$$x = \{ q_j \}$$

$$\psi = \prod (a_j \leq q_j) \wedge (q_j \leq b_j)$$

The analytical conditions of a transition firing will be considered without fixing the final value of q

variables. This approach is really useful to make a system specification.

2.3 Computing the next step

There are two different ways of reaching a state from a given one. The first solution is to use the evolution of associated variables. The second one is the transition firings. The following two definitions correspond to these two possibilities of evolution.

Definition 2.2: A state $E'(m', D', Val', X')$ is an accessible from another state $E(m, D, Val, X)$ according to associated variable evolution if and only if:

1- $m'=m$, and

2- $\forall j$ a token in p_j :

$$q'_i(j) = q_i(j) + \Delta q_i(j), \quad a_i \leq q'_i(j) \leq b_i$$

Where $[a_i, b_i]$ is the static interval of the place p_i . the possibility of reaching $q'_i(j)$ depends generally on the coupling with other q evolutions. This particular aspect is not presented here

Definition 2.3: A state $E'(m', D', Val', X')$ is accessible state from another state $E(m, D, Val, X)$ by firing transition t_i if and only if:

1- t_i is validated from E ,

2- $p \in P$,

$$m'(p) = m(p) - \text{Pre}(p, t_i) + \text{Post}(p, t_i),$$

$\text{Pre}(p, t_i)$ corresponds to the weight of the output arcs from p to t_i .

$\text{Post}(p, t_i)$ corresponds to the weight of the input arcs from t_i to p .

3 - Tokens that remain in the same place keep the same associated value between E and E' .

The newly created tokens take null values for the q counter associated to their new places. The value allocated to the token k' by Val is:

$$\text{Val}(k') = \text{Val}(k) + q(k),$$

where k is a token that is in an input place p_j of t_i and consumed to fire t_i .

The previous firing rule allows computing states and accessibility-relationships. The set of the firing sequences from an initial state specifies the PN behaviour as well as sets of accessible markings or validated firing sequences in the case of Autonomous PN.

P-time PN: a sub-class of ICPN

Actually, P-time PN may be defined as a sub-class of ICPN. It use the following restrictions:

The arrival set off IS application is restricted from $Q \cup \{-\infty, +\infty\} \times Q \cup \{-\infty, +\infty\}$ to $Q^+ \cup +\infty \times Q^+ \cup +\infty$.

The signification of the parameter associated to the tokens is the age. This restriction respects the ICPN definition.

The X application which provides the definition of parameter's evolution is strictly fixed, because it corresponds to the linear time evolution. Let us remark that this function match the monotonous criteria.

ICPN may be seen as a functional abstraction of P-time PN. The time is only a particular value in the domain of possible interpretations of parameters.

Theorem: structural mathematical properties of P-time PN can be extended to ICPN if and only if:

$$\forall i | 1 \leq i \leq n, n = \text{Card}(P), \text{mini}(q_i) \neq -\infty$$

Of course, the properties extension implies that the structure and the marking restriction are the same.

Proof: ICPN instances have all the same mathematical model. The only specificity of P-time PN is the restriction of the token associated parameter to positive rational numbers and the definition of the X application.

Consequently, any demonstration which do not use an assumption on X definition and which only use the marking and the net structure will be valid for any instance of ICPN.

There is only one condition to be fulfilled: the minimal values for the static intervals associated to places must not be equal to minus infinite. Actually, if this minimal value is negative and finite, a simple change of the origin on the axe of parameter values will shift the arrival set of IS intervals into positive values.

Nevertheless, the physical interpretation of the same property in each instance of model will depend on the physical meaning of parameters which are associated to the tokens.

3. USING A THEOREM OF P-TIME PN (COLLART 2002)

Let C_{t_1} and C_{t_2} be two different adjustments to instantiate associated values to pairs (place, token).

C_2 is the cycle value for C_{t_2} and C_1 is the cycle value for C_{t_1} .

p_i has an associated validity interval $[a_i, b_i]$.

We call $q_{i,C_{tj}}$ the effective value of the staying time of one token during a mono-periodic functioning according to an adjustment C_{tj} .

$At_{i,C_{tj}}$ is the margin available in p_i and allows to respect a product quality constraint or periodicity constraint. This margin characterizes C_{tj} .

$$At_{i,C_{tj}} = (q_{i,C_{tj}} - a_i).$$

There is a succession of states that respect staying time constraints for all tokens in the net. It allows moving from a state to another within mono-periodic functioning corresponding to C_{t_i} to a state that belongs to a mono-periodic functioning corresponding to C_{t_j} . The transition between C_{t_i} and C_{t_j} is noted $C_{t_{ij}}$.

dt_{ij} represents the total duration of the transient mode $C_{t_{ij}}$ [7].

Let SC_i the primary critical structure indexed by i and nc_i the number of tokens in SC_i . Rigorously, this magnitude must be a function of time.

Suppose that our workshop contains only primary critical structures. Call p_r the place modelling a constraint within a critical structure SC_r .

Theorem (Collart 2002):

$$(\forall At_{r,C_{t1}}, \exists C_1 | At_{r,C_{t1}} > 0) \Rightarrow \exists C_{t12}$$

$$nt_{12} = \lceil (C_2 - C_1) / \Delta C^* \rceil + 1$$

$$dt_{12} = n^* \cdot nt_{12} \cdot (C_1 + (\Delta C^* \cdot nt_{12}) / 2)$$

where:

$$\Delta C^* = \min_r (At_{r,C_{t1}} / nc_r).$$

$$n^* = \max_r (nc_r).$$

nt_{12} : represents the number of intermediate steps between the adjustments C_{t1} and C_{t2} . The intermediate step includes firing all the transitions of the net for n^* times.

nc_r : is the number of tokens in an elementary circuit or the difference of tokens between a synchronization transition and a parallelism one. It is quite normal in a dynamic system that an event requires multiple resources. The resulting synchronization of resources can be captured by transitions of the type shown in Fig 3.1. Here, t_4 is enabled only when each of P_3 and P_4 receives a token. The arrival of a token into each of the two places could be the result of a possibly complex sequence of operations elsewhere in the rest of the Petri net model. Essentially, transition t_1 models the joining operation. Parallel computing is a form of computation in which many instructions are carried out simultaneously, the transitions t_2 and t_3 are concurrent. Concurrency is an important attribute of

system interactions. Note that a necessary condition for transitions to be concurrent is the existence of a forking transition that deposits a token in two or more output places.

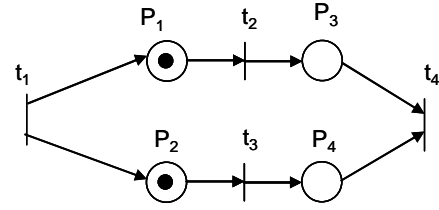


Fig 3.1: the typical characteristics in PN

The proof was been given in the case of P-Time PN. A functional abstraction allows one to generalize the property to ICPN.

The marking functions that depend on time have fixed initial values, henceforth.

The algorithm allowing the transition from adjustments C_{t1} to another C_{t2} is as follows.

Φ_{cd} :

$$SC_r = \{p_k, {}^{\circ}p_k, p_k^{\circ}\}_{k \in N^+ \cap [i,j]} \rightarrow SC_{r^*} = \{p_k, {}^{\circ}p_k, p_k^{\circ}, p_{r^*}\}_{k \in N^+ \cap [i,j]},$$

where:

$$a_k = \min(q_{k,C_{t1}}, q_{k,C_{t2}})$$

$$b_k = \max(q_{k,C_{t1}}, q_{k,C_{t2}})$$

$$p_{r^*}^{\circ} = p_j^{\circ}$$

$${}^{\circ}p_{r^*} = {}^{\circ}p_i$$

$$a_{r^*} = \sum_{k=i}^{k=j} q_{k,C_{t1}} - \sum_{k=i}^{k=j} M(p_k) \cdot C_d + nc_r \cdot \Delta C^*$$

$$b_{r^*} = \infty$$

Remark: Place p_{r^*} models the staying time constraints. It specifies the global variation of the staying time in a specific elementary circuit.

Let R be the expansion of the net into critical circuits SC_r .

The algorithm below [4] is applied in order to compute the set: $\sum_{k=i}^{k=j} \Delta q_k$

i) The first step: construction mode transitions

for $r=1$ to $\text{card } |R|$

$$\text{if } \left(\varepsilon \times \sum_{k=i}^{k=j} (q_{C_{t2}} - q_{C_{r^*}}) \geq \varepsilon \times nc_r \times \Delta C^* \right)$$

then

$$SC_r \leftarrow \Phi_{r^*}(SC_r)$$

else

$$\forall m \in [i, j], \begin{cases} q_{m,Ct1} < q_{m,Ct2} \\ q_{m,Ct_n^*} = q_{m,Ct2} \end{cases} \vee \begin{cases} q_{m,Ct1} > q_{m,Ct2} \\ \Delta q_m = 0 \end{cases}$$

end for

ii) The second step: Algorithm for calculating firing time in a Strongly Connected Event Graph

An algorithm computes the firing instants in $\phi_{1^*}(G)$ if $C_1 < C_2$ and in other cases:

• **Case where $C_1 < C_2$**

A graph G' is associated to a strongly Connected Graph events Related P-time G within a 1-periodic functioning with period equal C . The nodes of G' corresponds to transitions of G .

The arrows of G' are obtained from G 's places: two arrows are associated to each place p .

- the first arrow links $\bullet p$ to p^* , its ponderation is computed according to the next formula :

$$v_p = a_p - C.m_p$$

- the second links p^o to ${}^o p$, its ponderation is evaluated as follows:

$$v'_p = -b_p + C.m_p$$

- Choose a transition t_s , and associate $St_s(1) = 0$ to t_s

- Associate to each transition $t_u \in T$, $St_u(1) =$

$$\max_{l_{su}} \sum_{p \in l_{su}} v_p$$

• **Other case**

$$t_u \in T : St_u(1) =$$

$$\min_{l_{su}} \sum_{p \in l_{su}} v'_p$$

where l_{su} is an direct elementary path linking s to u .

In our case, the return constraint $[a_i, b_i]$ leads us to modify the algorithm (Collart 2003). In fact, this one supposes that there is no time constraint in places modelling robot comeback. The interval allocated to such a place looks like $[a_i, \infty]$

Calculating Waiting times:

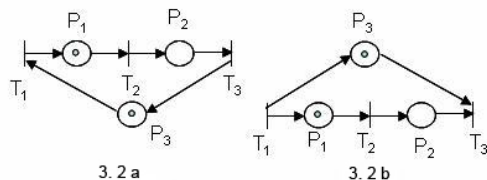


Figure 3.2: Waiting time

Consider the cases described in Figs 3.2a and 3.2b. Waiting times can be computed in two cases:

Case 1: Delay or C cycle time extension. A token reaches a transition T_e early. The problem is how to manage a part (P_1) that arrives early while a second part (P_2), to fulfil production, is not yet present. In this case, we check if there is a waiting margin for this part in P_2 . These margins are computed as follows:

$$\begin{aligned} At_i &= q_i - a_i : \text{for the "a" circuit} \\ At_i &= b_i - q_i : \text{for the "b" circuit} \end{aligned}$$

Case 2: Advance or C cycle time reduction. A token reaches a transition T_e late. In the same way we look for an available margin of this token, in order to make a synchronization with a token already existing in place P . This margin is given by the following relationships:

$$\begin{aligned} At_i &= b_i - q_i : \text{for the "a" circuit} \\ At_i &= q_i - a_i : \text{for the "b" circuit} \end{aligned}$$

Remark: When there is no token difference within places, this problem do not appear.

4. Application to regulating the weight of cigarettes.

The previous section ends with a theorem useful in the regulation of discrete processes working under time constraints. In this section, we extend the concept of p-time Petri nets to the interval constrained Petri nets. We try to apply our proposed net structure in the field of quality control of tobacco production, where time constraints cannot be ignored when discrete-event-driven models of such systems are established.

Definition 4.1: An ICPN model "Weight Value" is an ICPN model where the value associated with the couple (place, token) is interpreted as a weight.

Of course, it is not a question here to introduce a new sub-class of ICPN for every interpretation.

This tool ICPN "Weight Value" allows one to model a manufacturing process where the weight of the produced item must belong to a given interval.

This is, for example, the case of cigarette making workshop of tobacco industry described below.

As an example, we will use a proof that gives analytical conditions for the evolution from a cyclic functioning point to another within a P-time PN. After the presentation and modelling of the process, we will discuss the new significance of this theorem in the case of the ICPN "Weight Value" and its ensuing applications.

4.1 Workshop design

One of manufacturing systems where the respect of a produced item's weight arises is a cigarette making workshop. The process, as shown in Fig 4.1, makes a regular and homogenous tobacco pudding (endless cigarette): A beam of tobacco is enveloped by cigarette paper by means of an adhesive. The resulting pudding is cut up into segments corresponding to one cigarette in order to obtain a rough consumed unit (cigarette without filter).

Within this process, a weight interval constraint must be respected.

In fact, from a quality point of view a too heavy cigarette is difficult to draw and a too light one does not satisfy requirements.

The production of cigarettes consists of three steps as illustrated in Fig 4.1:

- preparation of a tobacco cut beam that will be setting to obtain a module (**m**)
- forming of a pudding with density (**d**) by enveloping the beam with cigarette paper.
- cutting up the pudding into (**l**) long rough consumed units. Henceforth, such unit is called simply cigarettes.

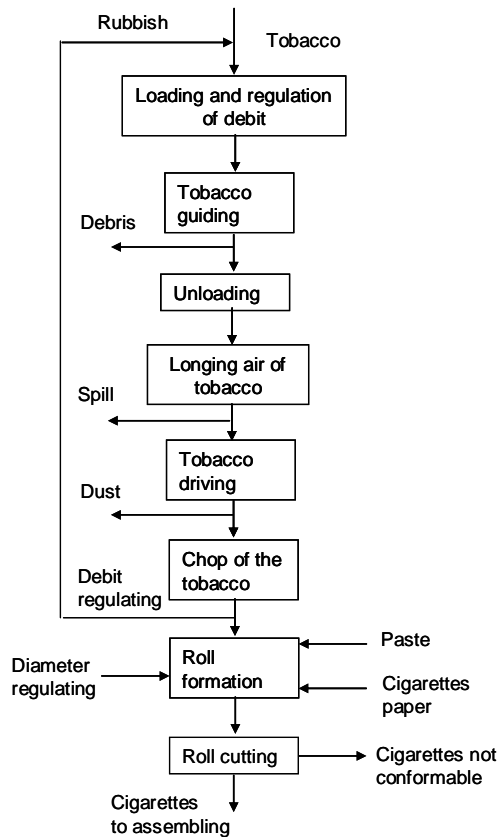


Fig 4.1: Process of manufacturing cigarettes

A cigarette may be compared to a cylinder with diameter m and length l . Thus, the weight of a cigarette can be expressed by the following relationship:

$$P = \frac{\pi}{4} \times m^2 \times l \times d \quad (4.1)$$

where :

- m : cigarette's module in mm with $m \in [m_{\min}, m_{\max}]$;
- l : cigarette's length in mm with $l \in [l_{\min}, l_{\max}]$;
- d : density in g/mm^3 with $d \in [d_{\min}, d_{\max}]$;

Note that these parameters (weight, module, length and density) are related. Obviously, the variation of one of these parameters provides a variation of the weight. When it is outside the validity range, the production has to be rejected or a machine blocking.

Our objective is to make sure that the permitted tolerance concerning cigarettes weight will be respected by controlling m , l and d parameters. It must belong to a predefined interval. The aim of the controller is to maintain the weight specification by changing the setting m , l and d parameters, whereas they have to remain in a validity interval:

$$P_{\min} \leq \frac{\pi}{4} \times m^2 \times l \times d \leq P_{\max} \quad (4.2)$$

4.2 Linearization around a reference state

Equation (4.1) is nonlinear. To resolve it we proceed by a linearization using a first order Taylorian development (in general, to make statistical analysis we only proceed to a first or second order development).

The four parameters taking part in (4.1) are not perfectly constants. They change around a medium position which corresponds to the stable state of the systems. Then we can assimilate equation 4.1 to a linear relationship by proceeding to one first order limited development and we can write growth of this function in the following linear form:

$$\Delta P \leq K_1 \times \Delta d + K_2 \times \Delta m + K_3 \times \Delta l \quad (4.3)$$

with:

$$\begin{aligned}
 K_1 &= (\pi/4) l_0 m_0^2 \\
 K_2 &= (\pi/2) l_0 m_0 d_0 \\
 K_3 &= (\pi/4) d_0 m_0^2 \\
 \Delta P &\in [P_{\min}, P_{\max}];
 \end{aligned}$$

p_0, m_0, c_0 and l_0 , respectively the values targets of the parameters p, m, c and l

4.3 Construction of model “Weight value”

The equation (4.3) can be represented by the ICPN shown in Fig 4.2. It called “Weight Value ”.

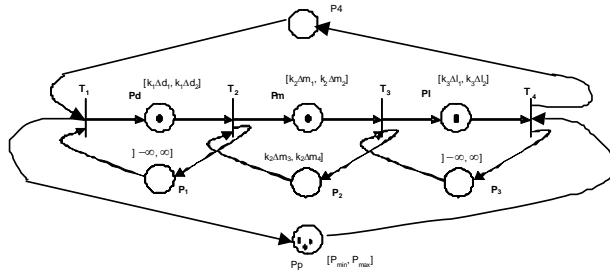


Fig 4.2: ICPN model “weight value”

The PN (Fig 4.2) represents equation 4.3 that models weight variations concerning the three parameters: density, module and length. Thus, the medium variation of circuits is not equal for all parameters. But, if we balance arrow weights, we could have obtained the same medium value for all circuits. This system’s invariant characterizes the weight conservation within the process.

The model in Fig 4.2 consists of:

- 8 places $P_d, P_m, P_l, P_p, P_1, P_2, P_3$ and P_4 .
- 4 Transitions T_1, T_2, T_3 and T_4
- 5 elementary circuits:
 - $\gamma_1 = T_1, P_d, T_2, P_1$
 - $\gamma_2 = T_2, P_m, T_3, P_2$
 - $\gamma_3 = T_3, P_l, T_4, P_3$
 - $\gamma_4 = T_1, P_p, T_4, P_4$
 - $\gamma_5 = T_1, P_d, T_2, P_m, T_3, P_1, T_4, P_4$

Places P_1 and P_3 are used to limit at one the number of tokens for a specific operation. As they have not other uses we will associate to them the interval $(-\infty, +\infty)$. They will not restrict, then, the net behaviour.

Place P_2 models the constraint characterizing module variation.

The production process corresponding to places P_d and P_m (density and module) is continuous. It is considered as a discrete one during our study. In fact, module and density measurements are done through specimens. We consider that twenty cigarettes can give us good representative and reliable information.

Density is subject to hazardous variations. Its distribution follows the normal distribution (based on production data). We will then compensate these lapses in acting on the module and length that are adjustable and depend on the state system. However, this regulation should include a coupling system that affects its dynamic behavior.

4.4 Condition theorem verifying

The cigarette production cell can be decomposed into critical structures. In fact, the succession of transformation operations constitutes a sequence. Elsewhere, limitations of physical stocks provoked by cyclicity constraints are modelled by the presence of combacks within the process. This leads to the appearance of elementary circuits.

Finally, constraints that concern weight are represented by a place that is parallel to the operating process model. There are, then, pair of paths that start with parallel transitions and end with a synchronization one. Here again, there is a conformity with the definition of primary critical structures. The considered workshop meets the application conditions of the theorem emitted previously.

In order to illustrate this theory, consider the model “Weight Value” in Fig 4.2. It models weight variation of a cigarette according to equation (4.3).

Mathematically, time PN and any ICPN (for example the ICPN “Weight Value”) have the same properties. However, the physical interpretation that must be given to the model is completely different.

Table 4.1 summarizes significance of different parameters that take part in both time PN and the ICPN “Weight Value”.

Table 4.1: Comparison of parameters P-time PN / ICPN model

| Parameter | P-Time PN signification | ICPN model |
|------------|--|---|
| C | Time Cycle | Weighing cycle for a piece |
| ΔC | Variation of time cycle | Variation of weight per cycle for a piece (compared to a reference) |
| Q | Effective time passed in a place | Weight added in this place |
| ΔQ | Variation of effective time | Variation of the added weight in place compared to a reference. |
| ai | Lower bound indicates the minimum time needed to execute the operation | Lower bound indicates the minimum weight added otherwise the quality of product is deteriorated |
| bi | The upper bound fixes the maximum time to not exceed | Upper bound indicates the maximum weight added otherwise the quality of product is deteriorated |
| M | Product, resource, constraint | Product, resource, constraint. |

The effective value of a parameter can be calculated with polynomial algorithms. This can be done because the above algorithm is only based on the structural properties of P-time Petri Net. In this case, it was shown that, under some particular assumptions, the property may be extended to ICPN (Collart 2002). In fact, the medium weight of cigarettes must be as near as possible from the target weight.

As it is impossible to check all cigarettes for high speed production, we proceed by selecting successively a quantity of cigarettes. Measurements are taken via this specimen in order to compute medium weight value and standard deviation [8].

When a deviation is detected, we look after a regulation that is able to manage the new value of the density and then the weight.

Figure 4.3 presents a basic functional block allowing to regulate weight: the micro-onde capture head, the weight regulation and the adjustment of tobacco cordon.

- The Microonde capture head gives metrological values to the weight regulation module.
- Le μP calculator determines the current weight of cigarettes and evaluates the production quantity. The cigarette weight is automatically controlled. In case of variations (A) compared to order (B) the weight controller WT react by computing a new order (D) which will affect the Ecreteur Disks. The positioning system POS performs the corresponding weight correction. If the real weight and the expected one do not coincide the regulation module computes a new position for a shop system. When the linear Transducer T indicates that this one has reached the new position, a new comparison between the effective weight and the expected one is carried out.

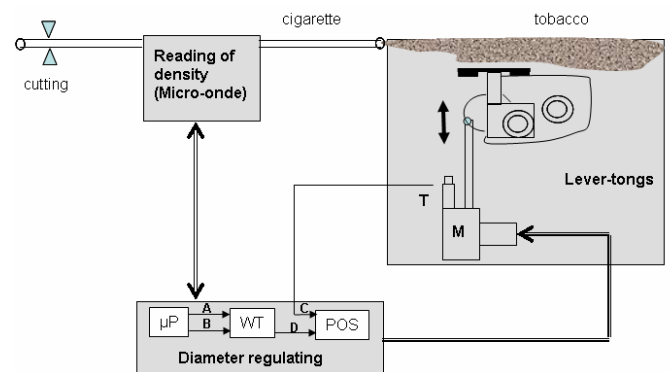


Fig 4.3: Structure of the regulatory system

A reversing relay controls the rotation sign of servomotor M in order to Ecreteur Disks (fig. 4.3) up or down according to the weight value in order to decrease or increase tobacco amount.

Deviation with target value is computed by studying a specimen of 20 cigarettes.

Under these conditions, the application of the theorem presented previously is exactly identical. So, there is a noticeable difference between the P-time model and the ICPN model "value weight" of our example: the "value weight" uses a differential of the first order around a reference point that allows us to linearize the pattern. Indeed, it is clear that in general, the calculation of cycle time in ICPN model is not linear. In this way, one can quantify the change in weight that occurs at different positions. With the help of this property one could therefore move within the space of valid solutions. Figure 4.4 represents the model "value weight" trucking Fig 4.2 with intervals of validity of each parameter.

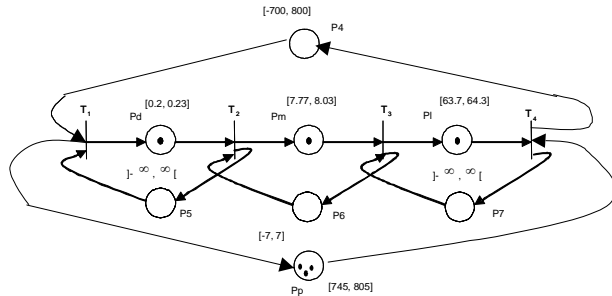


Fig 4.4: ICPN "weight value"

Suppose that the initial values of parameters (module, length and weight) are respectively:

$m=8$ mm, $l = 64$ mm and $P = 770$ mm.

The density of the sample production is $d = 0.21$.

Our objective is to adjust the weight to $P = 740$ g. Intermediary regulations allow to reach this objective as given in table 4.2.

Table 4.2: Settings intermediaries to a target values

| step | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---------------|-----|------|------|------|-----|------|------|-----|
| d=0.21 | | | | | | | | |
| <i>m</i> | 8 | 7.98 | 7.95 | 7.93 | 7.9 | 7.9 | 7.9 | 7.9 |
| <i>l</i> | 64 | 64 | 64 | 64 | 64 | 63.8 | 63.5 | 63 |
| <i>P</i> | 770 | 768 | 762 | 758 | 752 | 750 | 746 | 740 |
| <i>Atp</i> | 30 | 28 | 22 | 18 | 12 | 10 | 6 | 0 |
| <i>Atm</i> | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

In this table, *Atp* and *Atm* are respectively available margins in places Pp and Pm used to respect the product quality. These margins are specific to transitory regulations.

Suppose now that the density value is derived to take a new value $d = 0.22$. This obliges one to proceed to the other regulations in order to compensate this deviation and reach the target weight. Table 4.3 gives intermediate adjustments and the weight value we are able to reach.

Table 4.3: Settings intermediaries to compensate a density fluctuation

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---------------|------|------|------|------|------|-----|------|------|-----|------------|
| d=0.22 | | | | | | | | | | |
| 7.9 | 7.93 | 7.95 | 7.98 | 8 | 8 | 8 | 8.05 | 8.08 | 8.1 | 8.1 |
| 63.5 | 63.5 | 63.5 | 63.5 | 63.5 | 63.8 | 64 | 64 | 64 | 64 | 64.5 |
| 684 | 690 | 693 | 698 | 702 | 705 | 707 | 716 | 721 | 725 | 730 |
| 47 | 41 | 38 | 33 | 29 | 26 | 24 | 15 | 10 | 6 | 0 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

Consider now quantity variations in comparison to medium values representing targets values. Then we consider that values described in Fig. 4.4 are such that every step of the Qi are the values recorded in Table 4.2.

For example, for step 1, $m = 8$ mm with constraint on the module such as $m \in [7.9, 8.1]$, we find $\Delta m_1 = [-0.1, 0.1]$. A similar reasoning on all places for the first step gives the first column of table 4.4. We proceed in the same way for the remaining steps to obtain Table 4.4 is obtained.

Table 4.4: Variation of parameters relative to the target values

| steps | 1 | 2 | 3 | 4 | 5 | 6 |
|---------------|-------------|---------------|---------------|---------------|-----------|-------------|
| d=0.23 | | | | | | |
| Δm | [-0.1, 0.1] | [-0.08, 0.12] | [-0.05, 0.15] | [-0.06, 0.17] | [0, 0.2] | [0, 0.2] |
| Δl | [-1, 0] | [-1, 0] | [-1, 0] | [-1, 0] | [-1, 0] | [-0.8, 0.2] |
| ΔP | [-50, 4 0] | [-48, 52] | [-42, 48] | [-38, 52] | [-32, 58] | [-30, 60] |

4.5 Interpretation of results

A brief interpretation of weight constraint is given here. As the graph representing the system is strongly connected, the weighting associated to an elementary circuit corresponds to the value in mg relative to the target weight. Remember that we have proceeded to a first order limited development around this reference.

Theorem introduced in this section gives a condition to change cyclic instruction. Considering the slowly drifts of the process they can be anticipated. It is exactly the case of our study, if we consider variations of tobacco density that must be compensated by length or module corrections.

The theorem gives a guarantee concerning system reactivity: a maximal number of cycles is given to move from an adjustment to another. It will be possible, then, to compute a new instantiation of parameters (able to manage the new value of the density) and to move towards this new adjustment.

During this procedure, the respect of constraints is ensured. Finally, as constraint violations lead to reject some quantity of products present on machines, we have provided analytical tools able to reduce rejections. Within the studied production cell, all other machines work on the same speed of the critical machine. By reducing variations, production rates are proportionally brought up.

5. CONCLUSION

The ICPN model "Weight Value" concerns processes where the conformity of the finished product depends on the value of the weight by a produced unit. This value must belong to a certain validity interval. Outside this interval product is considered as rejection. To improve production performance within such a process, the control of the quality constraint must be able to adjust fluctuations that affect the system's entries. This adjustment has to be done through intermediate regulations that do not alter specifications. Regulations concern parameters that influence directly the weight. In our case parameters are density, module and length.

The proposed approach is portable since we have allowed attention to enable derivation of the ICPN introduced in this paper. Those PN are themselves a restriction of Abstract Marking PN that are clearly defined next to HLPN [9]. The functional abstraction made within ICPN allows taking into account varied linear constraints.

Otherwise, the tools used in HLPN-AM can be useful during a first step of scheduling and planning [10]. The generalization of the concept of P-time PN provides analytical tools useful to prove system's reactivity and regulate a critical quality.

One must note that an adjustment near maximal or minimal bounds might increase rejections rates in the case of noised entries. A multicriteria evaluation, in the context of workshop presenting interval validity constraints, could resolve such a problem.

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Lotfi NABLI received his Mastery of Sciences and DEA from ENSET- Tunis - Tunisia in 1989 and 1991 respectively. In 2000, he obtained his doctorate degree in Industrial automation: Automatic and Industrial computing from University of the sciences and the technologies of Lille France. He is currently Assistant professor of

Electrical Engineering at National School of Engineers of Monastir (ENIM) - Tunisia and a Master of conference candidate. His research interests include Modeling, Intelligente Control and Monitoring and command Manufactory systems.



Hedi Dhouibi received his Engineer degree at maintenance and DEA from National School of Engineers of Monastir (ENIM) - University of Center, Tunisia in 1997 and 1999 respectively. In 2005, he obtained his doctorate degree in Industrial automation:

Automatic and Industrial computing from University of the sciences and the technologies of Lille France. He is currently Assistant professor of Electrical Engineering at University of the sciences of Gabes Tunisia. His research interests include Modeling, Intelligente Control and Monitoring and command Manufactory systems.