

# A High Gain Observer and Sliding Mode Controller for an Autonomous Quadrotor Helicopter

M'hammed GUISSER and Hicham MEDROMI

*Abstract*- This paper deals with the design of a high gain observer and sliding mode controller for a remotely controlled quadrotor vertical take off and landing aircraft. Under the restriction that only the inertial coordinates and yaw angle are available for measurement, a high gain observer is designed which allows estimating on-line the roll and pitching angles as well as all the linear and angular velocities of the vehicle. The problem is solved in two steps: first, a high gain observer design is carried out by means a change of coordinates of the quadrotor system. Second, a dynamic sliding mode controller is proposed using the state estimate of the quadrotor helicopter. We show the asymptotic stability of the couple observer-controller. Finally, computer simulations are developed for showing the performance of the proposed observer-based control.

*Index Terms*—Helicopter dynamic, State observer, Sliding mode control, Observer-based control.

## 1. INTRODUCTION

As their application potential both in the military and industrial sector strongly increases, miniature unmanned aerial vehicles (UAV) constantly gain in interest among the research community. Mostly used for surveillance and inspection roles, building exploration or missions in inaccessible or dangerous environments, the easy handling of the UAV by an operator without hours of training is primordial. In order to develop a reliable assisted remote control or guarantee the capability of a stable autonomous flight, the development of simple and robust control laws, stabilizing the UAV even in the presence of wind and turbulences becomes more and more important. This idea motivates many studies towards the quadrotor helicopter [1], [2], [3]. Sliding mode controller (SMC) systems are very well known for their robustness against disturbances, uncertainty, unmodelled dynamics, and invariance during the sliding mode. Conceptually, the controller design in this framework is based on the nominal representation of the system about which the bounds of the uncertainties are assumed to be available. The technique is first introduced by Sira-Ramirez et al [4] for a miniature radio-control Helicopter. Their work applies it to altitude stabilization only. The work [5] proposes a control law based on a sliding mode approach in order to stabilize the rotation subsystem of the quadrotor helicopter. The current SMC design problem is involved with coupled and highly nonlinear dynamics, noisy observations and demanding performance requirements [6].

However, only a few of the control strategies have been tested experimentally. One of the main obstacles to perform real-time experiments is that the control law needs

to feedback all the quadrotor states, but not all of them are available for measuring [7]. Indeed, even if all the state measurements are possible they are typically corrupted by noise. Moreover, the increased number of sensors makes the overall system more complex in implementation and expensive in realization [8]. In order to reduce the number of sensors required for control design, the papers [9], [10], present the sliding mode and high-order sliding mode respectively like an observer [9] and [10] in order to estimate the unmeasured states and the effects of the external disturbances such as wind and noise. However, the stability of the couple observer-controller is only tested in simulation.

The main contributions of this work are: first, a high gain observer is synthesized. It needs only the inertial coordinates and yaw angle to estimate the roll and pitch angles, and all velocities. This observer has only one parameter to tune and can be used in the feedback loop because it is simple to implement and guaranteed to be stable. Second, a control algorithm based on sliding mode technique that ensures the asymptotic stability and desired tracking trajectories. It is designed as if all states were measured; then the high gain observer estimates are used instead of the true states. Finally, a kind of separation principle is studied in order to guarantee the stability of the closed-loop quadrotor system with the couple observer-controller, when the actual states are replaced by their estimates. The performance of the controller-observer is illustrated in a simulation study that takes into account the measurement noise.

This paper presents the following results: in the second part, models for the propulsion system and the flight dynamic of the UAV are presented. In the third part, a high gain observer is proposed to estimate the unmeasured states. In the fourth part, a sliding mode controller is synthesized to stabilize the position and attitude of the quadrotor helicopter. In the fifth part, a separation principle is proved to ensure the convergence of the global closed loop system. Finally, simulation results and conclusion are given at the end of the paper.

## 2. QUADROTOR DYNAMIC MODELLING

The quadrotor helicopter is a vertical take off and landing vehicle ideally suited for stationary and quasi-stationary flight conditions. The control of the quadrotor is achieved by the differential control of the thrust generated by each propeller. Up/down motion is controlled by collectively increasing or decreasing the thrust of all four motors. The thrust difference between the forward and rear rotors creates a pitch torque inducing translation

M. Guisser and H Medromi are with Dept. of Electrical Engineering, University, Hassan II Ain Chock, ENSEM, BP. 8118, Oasis Casablanca, MOROCCO (e-mail: mguisser@yahoo.fr and hmedromi@yahoo.fr).

forward/rear motion. In the same way, the left/right translational motion is obtained by the differential thrust of the right and the left rotors. Consider the yaw control. When a propeller turns, it has to overcome air resistance. The reactive torque acts on the blades in the opposite direction to the rotation. In the quadrotor, both sets of front-rear and left-right motors turn in opposite directions. Moreover, the reactive torque is essentially a function of the propeller rotational velocity. Consequently, controlling the quadrotor yaw is equivalent to controlling the sum of reactive torques. As long as all rotors produce the same reactive torque (all rotors turn at the same speed), the sum of all reactive torques is zero and there is no yaw motion. If one set of rotors increases its speed, the induced torque causes the quadrotor to rotate in the direction of the induced torque.

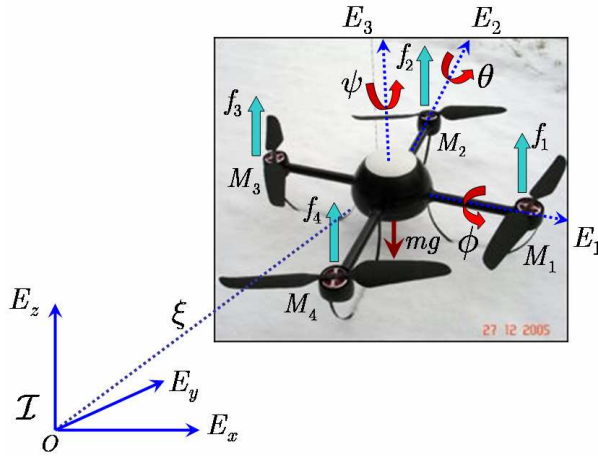


Fig. 1. General view of the quadrotor rotorcraft system.

Let  $\mathcal{I} = (E_x, E_y, E_z)$  denote an inertial frame attached to the earth, relative to a fixed origin, and  $\mathcal{B} = (E_1, E_2, E_3)$  denote a body fixed frame attached to the centre of mass of the vehicle as shown in Figure 1.

The position  $\xi = (x, y, z)$  of the UAV is given by the position of the origin of the body-fixed frame relative to the origin of the inertial frame. The orientation of the UAV in space is described by the Euler angles  $\eta = (\phi, \theta, \psi)$ . These angles are called the roll ( $-\pi/2 < \phi < \pi/2$ ), pitch ( $-\pi/2 < \theta < \pi/2$ ) and yaw ( $-\pi < \psi < \pi$ ), respectively. The representation of the rotation matrix  $R$  is based on the following rotation order: the first rotation with the angle  $\phi$  around the x-axis, the second rotation with the angle  $\theta$  around the new y-axis and the third rotation with the angle  $\psi$  around the new z-axis.

$$R \doteq R(\eta) = \begin{pmatrix} c_\psi c_\theta & -s_\psi c_\theta & s_\theta \\ c_\psi s_\theta s_\phi + s_\psi c_\phi & -s_\psi s_\theta s_\phi + c_\psi c_\phi & -c_\theta s_\phi \\ -c_\psi s_\theta c_\phi + s_\psi s_\phi & s_\psi s_\theta c_\phi + c_\psi s_\phi & c_\theta c_\phi \end{pmatrix}$$

where the following shorthand notations for trigonometric functions are used:  $c_\mu = \cos \mu$  and  $s_\mu = \sin \mu$ .

The rotation matrix of a quadrotor has been developed [21], [22] but in other manner, where the rotation order is: yaw, pitch and roll respectively. Our representation permits to obtain a practical quadrotor model that simplifies the synthesis of the controller and observer.

Let  $v = \dot{\xi} = (v_x, v_y, v_z)$  (resp.  $\Omega = (\Omega_1, \Omega_2, \Omega_3)$ ) denote the linear (resp. angular) velocity of centre of mass expressed in the inertial frame (resp. body fixed frame).

The angular rotation velocities  $\Omega$  in the body-fixed frame can be obtained with respect to the angular rotation velocities  $\dot{\eta}$  in the Euler coordinates:

$$\Omega = \begin{pmatrix} 1 & 0 & \sin \theta \\ 0 & \cos \phi & -\cos \theta \sin \phi \\ 0 & \sin \phi & \cos \theta \cos \phi \end{pmatrix} \dot{\eta} = W(\eta) \dot{\eta}$$

In the case when a quadrotor performs many angular motions of low amplitude where the roll angle  $\phi$  and pitch angle  $\theta$  are small, the angular velocity  $\Omega$  can be assimilated to  $\dot{\eta} = (\dot{\phi}, \dot{\theta}, \dot{\psi})$ .

The reactive torque generated, in free air, by the rotor  $i \in \{1, 2, 3, 4\}$  due to rotor drag is given by  $Q_i = \kappa \omega_i^2$ , and the total thrust generated by the four rotors is given by:

$$T = \sum_{i=1}^4 \|f_i\| = b \sum_{i=1}^4 \omega_i^2, \text{ where } f_i = b \omega_i^2 E_3 \text{ is the lift}$$

generated by the rotor  $i$  in free air, and  $\kappa > 0$  and  $b > 0$  are two parameters depending on the density of air, radius, shape and pitch angle of the blade [21].

The airframe torques generated by the rotors are denoted by  $\tau = (\tau_\phi, \tau_\theta, \tau_\psi)$ , with

$$\begin{aligned} \tau_\phi &= (f_2 - f_4)d; \\ \tau_\theta &= (f_3 - f_1)d \\ \tau_\psi &= (f_2 + f_4 - f_1 - f_3)\kappa, \end{aligned}$$

where  $d$  represents the distance from the rotors to the centre of gravity of the quadrotor helicopter.

In the sequel, the control inputs are denoted by the vector  $u = [u_1, u_2, u_3, u_4]^T = [T, \tau_\phi, \tau_\theta, \tau_\psi]^T \in \mathbb{R}^4$ , the measured and the controlled outputs are denoted by the vector  $Y = [Y_1, Y_2, Y_3, Y_4]^T = [x, y, z, \psi]^T \in \mathbb{R}^4$ . Define the state variables  $X = [\xi, \dot{\xi}, \eta, \dot{\eta}]^T \in \mathbb{R}^{12}$ .

The dynamical model of the quadrotor helicopter is given as follows:

$$\left\{ \begin{array}{l} \ddot{\xi} = \begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix} + \begin{pmatrix} \sin \theta \\ -\cos \theta \sin \phi \\ \cos \theta \cos \phi \end{pmatrix} \frac{u_1}{m} \\ \ddot{ij} = \begin{pmatrix} a_1 \dot{\theta} \dot{\psi} + b_1 u_2 \\ a_2 \dot{\phi} \dot{\psi} + b_2 u_3 \\ a_3 \dot{\theta} \dot{\phi} + b_3 u_4 \end{pmatrix} \end{array} \right. \quad (1)$$

where  $m$  is the mass of the quadrotor system. The parameters  $a_i$  and  $b_i$  are given by:

$$\begin{aligned} a_1 &= (I_y - I_z) / I_x; & b_1 &= 1 / I_x \\ a_2 &= (I_z - I_x) / I_y; & b_2 &= 1 / I_y \\ a_3 &= (I_x - I_y) / I_z; & b_3 &= 1 / I_z \end{aligned}$$

where  $I_x$ ,  $I_y$  and  $I_z$  are the inertia elements of the quadrotor with respect to  $\mathcal{B}$ .  $g$  is the gravity acceleration.

### 3. HIGH GAIN OBSERVER DESIGN

In this section, a high gain observer is designed, with local exponential error convergence; it needs only the Cartesian coordinates and yaw angle to estimate the roll and pitch angles, and all velocities. The observer is synthesized from a canonical form that characterizes the class of systems with locally regular inputs and satisfying some regularity assumptions [11], [12]. After a change of coordinates (diffeomorphism), the quadrotor system can be transformed into the canonical form composed of a state affine linear part, used for the synthesis of the observer, and again a nonlinear controlled part which will be hidden by a high gain synthesis [13], [14]. The gain of the proposed observer is issued from a differential Lyapunov equation.

The  $(x, y)$  position can be obtained by a differential global positioning system (DGPS) that provides high-accuracy position and velocity information [8]. The vertical direction  $z$  can be obtained by a sonar altimeter (based on ultrasonic transducer) that is employed to provide altitude information at a reasonable accuracy. In [7], for the inertial coordinate measurement a positioning system based on ultrasonic signals and Newton method is used. A digital compass is used to establish the helicopter yaw angle  $\psi$  to about  $\pm 1^\circ$  [8]. While the navigation sensors and associated inner-loop control software are able to have the helicopter take-off, land, and fly from waypoint to waypoint, the UAV must be capable of establishing these waypoints. Since the competition environment is dynamic, this must be done in real time.

The dynamical model of the quadrotor helicopter can be written as:

$$\left\{ \begin{array}{l} \ddot{x} = \sin \theta \frac{u_1}{m} \\ \ddot{y} = -\cos \theta \sin \phi \frac{u_1}{m} \\ \ddot{z} = -g + \cos \theta \cos \phi \frac{u_1}{m} \\ \ddot{\phi} = a_1 \dot{\theta} \dot{\psi} + b_1 u_2 \\ \ddot{\theta} = a_2 \dot{\phi} \dot{\psi} + b_2 u_3 \\ \ddot{\psi} = a_3 \dot{\theta} \dot{\phi} + b_3 u_4 \end{array} \right. \quad (2)$$

Let us consider the change of coordinates  $\Phi : K \subset \mathbb{R}^{12} \rightarrow M \subset \mathbb{R}^{12}$ ,  $\mathcal{Z} = \Phi(X)$ ,  $K$  and  $M$  are a compact sets of  $\mathbb{R}^{12}$ , such that the quadrotor system (2) in the new system of coordinates can be transformed into the canonical form, where the state affine linear part depends on the thrust input  $u_1$ . The change of coordinates is defined as:

$\mathcal{Z} = [\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4]^T$ ,  $\mathcal{Z}_j = [\mathcal{Z}_{j1}, \mathcal{Z}_{j2}, \mathcal{Z}_{j3}, \mathcal{Z}_{j4}]^T$ ,  $j = 1, 2$  and  $\mathcal{Z}_i = [\mathcal{Z}_{i1}, \mathcal{Z}_{i2}]^T$ ,  $i = 3, 4$ . The new system of coordinates is given explicitly by:

$$\begin{aligned} \mathcal{Z}_{11} &= x; & \mathcal{Z}_{21} &= y \\ \mathcal{Z}_{12} &= \dot{x}; & \mathcal{Z}_{22} &= \dot{y} \\ \mathcal{Z}_{13} &= \sin \theta; & \mathcal{Z}_{23} &= -\cos \theta \sin \phi \\ \mathcal{Z}_{14} &= \dot{\theta} \cos \theta; & \mathcal{Z}_{24} &= \dot{\theta} \sin \theta \sin \phi - \dot{\phi} \cos \theta \cos \phi \\ \mathcal{Z}_{31} &= z; & \mathcal{Z}_{41} &= \psi \\ \mathcal{Z}_{32} &= \dot{z}; & \mathcal{Z}_{42} &= \dot{\psi} \end{aligned}$$

Indeed, let  $Y_1 = x$  be the first measured output. The non-measurable signals can be obtained by successive differentiation. Let  $\mathcal{Z}_1 = [\mathcal{Z}_{11}, \mathcal{Z}_{12}, \mathcal{Z}_{13}, \mathcal{Z}_{14}]^T$ , where  $\mathcal{Z}_{11} = Y_1 = x$ . One has:

$$\begin{aligned} \dot{\mathcal{Z}}_{11} &= \dot{x} = \mathcal{Z}_{12}, \\ \dot{\mathcal{Z}}_{12} &= \ddot{x} = \sin \theta \frac{u_1}{m} = \mathcal{Z}_{13} \frac{u_1}{m}, \\ \dot{\mathcal{Z}}_{13} &= \dot{\theta} \cos \theta = \mathcal{Z}_{14}, \end{aligned}$$

$$\begin{aligned} \dot{\mathcal{Z}}_{14} &= \ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta = (a_2 \dot{\phi} \dot{\psi} + b_2 u_3) \cos \theta - \dot{\theta}^2 \sin \theta \\ &= \varphi_{14}(\mathcal{Z}, u) \end{aligned}$$

The first new system of coordinates associated to the output  $Y_1$  can be written under the following canonical form:

$$\left\{ \begin{array}{l} \dot{\mathcal{Z}}_1 = A_1(u) \mathcal{Z}_1 + \varphi_1(\mathcal{Z}, u) \\ Y_1 = C_1 \mathcal{Z}_1 \end{array} \right. \quad (3)$$

$$\text{where } A_1(u) = \begin{bmatrix} 0 & \frac{u_1}{m} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \quad \varphi_1(\mathcal{Z}, u) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \varphi_{14}(\mathcal{Z}, u) \end{bmatrix};$$

$$\text{and } C_1 = [1 \ 0 \ 0 \ 0].$$

The second new system of coordinates associated to the output  $Y_2 = y$  can be written under the following canonical form:

$$\begin{cases} \dot{\mathcal{Z}}_2 = A_2(u)\mathcal{Z}_2 + \varphi_2(\mathcal{Z}, u) \\ Y_2 = C_2\mathcal{Z}_2 \end{cases} \quad (4)$$

where  $A_2(u) = A_1(u)$ ,  $C_2 = C_1$  and  $\varphi_2(\mathcal{Z}, u) = [0 \ 0 \ 0 \ \varphi_{24}(\mathcal{Z}, u)]^T$ , with:

$$\begin{aligned} \varphi_{24}(\mathcal{Z}, u) = & (a_2\dot{\phi}\dot{\psi} + b_2u_3) \sin \theta \sin \phi + \dot{\theta}^2 \cos \theta \sin \phi + \\ & 2\dot{\phi}\dot{\theta} \sin \theta \cos \phi - (a_1\dot{\theta}\dot{\psi} + b_1u_2) \cos \theta \cos \phi + \\ & \dot{\phi}^2 \cos \theta \sin \phi \end{aligned}$$

An equivalent procedure is to be applied to the two remaining outputs  $Y_3 = z$  and  $Y_4 = \psi$

$$\begin{cases} \dot{\mathcal{Z}}_i = A_i\mathcal{Z}_i + \varphi_i(\mathcal{Z}, u) \\ Y_i = C_i\mathcal{Z}_i \end{cases} \quad (5)$$

where  $A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ ;  $C_i = [1 \ 0]$ , for  $i = 3, 4$  and

$$\varphi_i(\mathcal{Z}, u) = [0 \ \varphi_{i2}(\mathcal{Z}, u)]^T$$

with:

$$\varphi_{32}(\mathcal{Z}, u) = -g + \cos \theta \cos \phi \frac{u_1}{m}$$

$$\varphi_{42}(\mathcal{Z}, u) = a_3\dot{\phi}\dot{\theta} + b_3u_4$$

The original coordinates of the quadrotor helicopter are given by  $X = \Phi^{-1}(\mathcal{Z})$ :

$$x = \mathcal{Z}_{11}; \quad y = \mathcal{Z}_{21}$$

$$\dot{x} = \mathcal{Z}_{12}; \quad \dot{y} = \mathcal{Z}_{22}$$

$$\theta = \arcsin(\mathcal{Z}_{13}); \quad \phi = \arcsin\left(-\frac{\mathcal{Z}_{23}}{\cos \theta}\right)$$

$$\dot{\theta} = \frac{\mathcal{Z}_{14}}{\cos \theta}; \quad \dot{\phi} = \frac{\dot{\theta} \sin \theta \sin \phi - \mathcal{Z}_{24}}{\cos \theta \cos \phi}$$

$$z = \mathcal{Z}_{31}; \quad \psi = \mathcal{Z}_{41}$$

$$\dot{z} = \mathcal{Z}_{32}; \quad \dot{\psi} = \mathcal{Z}_{42}$$

The  $X$ -coordinates of the quadrotor system are transformed by means of the diffeomorphism  $\Phi$  into a condensed form of the new  $\mathcal{Z}$ -coordinates:

$$\begin{cases} \dot{\mathcal{Z}} = A(u)\mathcal{Z} + \varphi(\mathcal{Z}, u) \\ Y = C\mathcal{Z} \end{cases} \quad (6)$$

where

$A(u) = \text{diag}(A_1(u), A_2(u), A_3, A_4)$ ,  $\varphi = [\varphi_1 \ \varphi_2 \ \varphi_3 \ \varphi_4]^T$  and  $C = \text{diag}(C_1, C_2, C_3, C_4)$ . As long as  $u_1 \neq 0$ , it is clear that the system (6) is observable, since the admissible thrust input  $u_1$  is never to be zero while the quadrotor helicopter is in hover.

One assumes that system (6) satisfies the following Lipschitz assumption:

**Assumption 1:**  $\varphi(\mathcal{Z}, u)$  is a locally Lipschitz nonlinear function with respect to  $\mathcal{Z}$  uniformly to  $u$ , we have:

$\forall \mathcal{Z}; \bar{\mathcal{Z}} \in M \subset \mathbb{R}^{12}; \forall u \in \mathcal{U} \subset \mathbb{R}^4, \exists \mu_\varphi > 0$ , such that:

$$\|\varphi(\mathcal{Z}, u) - \varphi(\bar{\mathcal{Z}}, u)\| \leq \mu_\varphi \|\mathcal{Z} - \bar{\mathcal{Z}}\|$$

where  $\mathcal{U}$  is a bounded set of  $\mathbb{R}^4$ ,  $\mu_\varphi$  is the Lipschitz constant depending on the upper bound of the admissible control  $u$  such that  $\sup_{t \geq 0} \|u(t)\| < \infty$ .

A candidate observer for the transformed quadrotor system (6) is described by the following dynamical system:

$$\begin{cases} \dot{\hat{\mathcal{Z}}} = A(u)\hat{\mathcal{Z}} + \varphi(\hat{\mathcal{Z}}, u) - S^{-1}C^T(C\hat{\mathcal{Z}} - Y) \\ \dot{Y} = CY \end{cases} \quad (7)$$

where  $\hat{\mathcal{Z}} \in M$  is the estimated state,  $y$  and  $u$  are respectively the output and input of the transformed quadrotor system (6). The estimation of the state variables  $X = [\xi, \dot{\xi}, \eta, \dot{\eta}]^T$  can be found from the transformation equation  $\hat{X} = \Phi^{-1}(\hat{\mathcal{Z}})$ .  $S = \text{diag}(S_1, S_2, S_3, S_4)$  is a symmetric positive definite matrix and a solution of the differential Lyapunov equation:

$$\dot{S} = -\Theta S - A^T(u)S - SA(u) + C^T C \quad (8)$$

with  $\Theta > 0$  is a design parameter chosen sufficiently large and  $S_0 = S(0)$  being the initial condition for the solution  $S(t)$ . Notice that for any bounded admissible control  $u(t)$  the matrix  $S(t)$  is bounded [15], [16].

#### 4. SLIDING MODE CONTROL DESIGN

In this section, the purpose is to design a sliding mode controller (SMC). The basic SMC design procedure in our case is performed in two steps. Firstly, the choice of sliding surface  $\sigma$  according to the tracking error, while the second step is the design of a Lyapunov function that can satisfy the necessary sliding condition  $\sigma\dot{\sigma} < 0$ . During this sliding regime, the closed loop control system becomes insensitive to external perturbation signals, modelling errors and parameter variations.

The objective of the control is to track desired smooth trajectories  $\xi_d = (x_d, y_d, z_d)$  and  $\psi_d$  that represent the translational motion and yaw rotation, respectively for which the tracking errors  $\xi - \xi_d$  and  $\psi - \psi_d$  converge asymptotically to zero. The SMC proposed is based on the input-output decoupling technique. In order to render the

quadrotor system completely linearizable, the thrust input  $u_1$  has been delayed by a double integrator. The other control signals remain unchanged. This dynamic extension more naturally allows tradeoff between the various control objectives of the generic trajectory tracking problem. Then the dynamic extension is given by

$$\ddot{u}_1 = \bar{u}_1 \tag{9}$$

where the actual control  $u_1$  and its first derivative  $\dot{u}_1$  become internal variables of a dynamic controller. The rotational dynamic can be linearized with the static feedback control laws. thus we have:

$$\begin{aligned} \ddot{\phi} &= a_1 \dot{\theta} \dot{\psi} + b_1 u_2 = \bar{u}_2 \\ \ddot{\theta} &= a_2 \dot{\phi} \dot{\psi} + b_2 u_3 = \bar{u}_3 \\ \ddot{\psi} &= a_3 \dot{\theta} \dot{\phi} + b_3 u_4 = \bar{u}_4 \end{aligned} \tag{10}$$

Using the input–output feedback linearization procedure of the position  $\xi$  that has the relative degree of 4 with respect to the control input [18], [19], we have the third and fourth time derivative of the position  $\xi$  :

$$\xi^{(3)} = \begin{pmatrix} \dot{\theta} c_\theta \\ \dot{\theta} s_\theta s_\phi - \dot{\phi} c_\theta c_\phi \\ -\dot{\theta} s_\theta c_\phi - \dot{\phi} c_\theta s_\phi \end{pmatrix} \frac{u_1}{m} + \begin{pmatrix} s_\theta \\ -c_\theta s_\phi \\ c_\theta c_\phi \end{pmatrix} \frac{\dot{u}_1}{m} \tag{11}$$

$$\xi^{(4)} = \beta(X, u_1) \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \end{pmatrix} + \alpha_1(X) u_1 + \alpha_2(X) \dot{u}_1 \tag{12}$$

where  $\beta(X, u_1)$  ,  $\alpha_1(X)$  and  $\alpha_2(X)$  are computed as follows:

$$\begin{aligned} \beta(X, u_1) &= \frac{1}{m} \begin{bmatrix} s_\theta & u_1 c_\theta & 0 \\ -c_\theta s_\phi & u_1 s_\theta s_\phi & -u_1 c_\theta c_\phi \\ c_\theta c_\phi & -u_1 s_\theta c_\phi & -u_1 c_\theta s_\phi \end{bmatrix}, \\ \alpha_1(X) &= \frac{1}{m} \begin{pmatrix} -\dot{\theta}^2 s_\theta \\ \dot{\theta}^2 c_\theta s_\phi + 2\dot{\phi}\dot{\theta}s_\theta c_\phi + \dot{\phi}^2 c_\theta s_\phi \\ -\dot{\theta}^2 c_\theta c_\phi + 2\dot{\phi}\dot{\theta}s_\theta s_\phi - \dot{\phi}^2 c_\theta c_\phi \end{pmatrix}, \\ \alpha_2(X) &= \frac{2}{m} \begin{pmatrix} \dot{\theta} c_\theta \\ \dot{\theta} s_\theta s_\phi - \dot{\phi} c_\theta c_\phi \\ -\dot{\theta} s_\theta c_\phi - \dot{\phi} c_\theta s_\phi \end{pmatrix} \end{aligned}$$

An equivalent procedure is to be applied to the yaw attitude. The second time derivative of  $\psi$  is given by:

$$\ddot{\psi} = \bar{u}_4$$

If the thrust input  $u_1 \neq 0$  and the pitch angle  $-\pi/2 < \theta < \pi/2$  are satisfied, then matrix  $\beta(X, u_1)$  is not singular. Therefore, the control inputs of the quadrotor

helicopter are computed by means of a sliding mode approach.

The design problem is to enforce the behavior of the system states towards the desired trajectories, which are known. Denote the tracking errors by  $\varepsilon_\xi = \xi - \xi_d$  and  $\varepsilon_\psi = \psi - \psi_d$ .

The crux of the SMC scheme is the definition of a sliding manifold along which the sliding motion is to take place. These sliding surfaces are denoted by  $\sigma_\xi = (\sigma_x, \sigma_y, \sigma_z)$  and  $\sigma_\psi$ . These quantities are defined as the functions of the tracking errors such that:

$$\sigma_\xi = \ddot{\varepsilon}_\xi + \lambda_3 \dot{\varepsilon}_\xi + \lambda_2 \varepsilon_\xi + \lambda_1 \varepsilon_\xi \tag{13}$$

$$\sigma_\psi = \dot{\varepsilon}_\psi + \lambda_4 \varepsilon_\psi \tag{14}$$

where  $\lambda_1, \lambda_2, \lambda_3$ ;  $j = 1, 2, 3$  are positive elements chosen such that the polynomial  $P(p) = p^3 + \lambda_3 p^2 + \lambda_2 p + \lambda_1$  is Hurwitz and  $\lambda_4$  is a positive constant.

If a control law enforces the trajectories in the phase space such that  $\sigma = (\sigma_\xi, \sigma_\psi) = 0$  holds true, then the tracking errors converge asymptotically to origin as prescribed by  $\ddot{\varepsilon}_\xi + \lambda_3 \dot{\varepsilon}_\xi + \lambda_2 \varepsilon_\xi + \lambda_1 \varepsilon_\xi = 0$  and  $\dot{\varepsilon}_\psi + \lambda_4 \varepsilon_\psi = 0$ .

In order to demonstrate stability, we adopt the Lyapunov candidate functions given as:

$$V_\xi = \frac{1}{2} \sigma_\xi^T \sigma_\xi \tag{15}$$

$$V_\psi = \frac{1}{2} \sigma_\psi^2 \tag{16}$$

The time derivative of the Lyapunov functions in (15) and (16) can be computed as follows

$$\dot{V}_\xi = \sigma_\xi^T \dot{\sigma}_\xi = \sigma_\xi^T (\dot{\varepsilon}_\xi^{(4)} + \lambda_3 \ddot{\varepsilon}_\xi + \lambda_2 \dot{\varepsilon}_\xi + \lambda_1 \varepsilon_\xi) \tag{17}$$

$$\dot{V}_\psi = \sigma_\psi \dot{\sigma}_\psi = \sigma_\psi (\ddot{\varepsilon}_\psi + \lambda_4 \dot{\varepsilon}_\psi) \tag{18}$$

Therefore,

$$\dot{V}_\xi = \sigma_\xi^T (\xi^{(4)} - \xi_d^{(4)} + \lambda_3 \ddot{\varepsilon}_\xi + \lambda_2 \dot{\varepsilon}_\xi + \lambda_1 \varepsilon_\xi) \tag{19}$$

$$\dot{V}_\psi = \sigma_\psi (\ddot{\psi} - \ddot{\psi}_d + \lambda_4 \dot{\varepsilon}_\psi) \tag{20}$$

we would like to have  $\dot{V}_\xi = \sigma_\xi^T \dot{\sigma}_\xi < 0$  and  $\dot{V}_\psi = \sigma_\psi \dot{\sigma}_\psi < 0$ . Dropping the arguments  $\dot{\sigma}_\xi$  and  $\dot{\sigma}_\psi$  to  $-k_\xi \text{sign}(\sigma_\xi)$  and  $-k_\psi \text{sign}(\sigma_\psi)$  respectively, with  $k_\xi$  and  $k_\psi$  being positive parameters. We have:

$$\dot{\sigma}_\xi = -k_\xi \text{sign}(\sigma_\xi) \tag{21}$$

$$\dot{\sigma}_\psi = -k_\psi \text{sign}(\sigma_\psi) \tag{22}$$

which ensures that  $\dot{V}_\xi = -k_\xi \|\sigma_\xi\|$  and  $\dot{V}_\psi = -k_\psi |\sigma_\psi|$  are achieved in the closed loop. Then, the necessary sliding condition is verified and Lyapunov stability is guaranteed.

Equations (21) and (22) can be solved in order to determine the control variables. We obtain the following

controllers:

$$\begin{aligned} [\bar{u}_1 \ \bar{u}_2 \ \bar{u}_3]^T &= \beta^{-1}(X, u_1)[\xi_d^{(4)} - \alpha_1(X)u_1 - \alpha_2(X)\dot{u}_1 \\ &\quad - \lambda_3\ddot{\xi} - \lambda_2\dot{\xi} - \lambda_1\xi - k_\xi \text{sign}(\sigma_\xi)] \end{aligned} \quad (23)$$

$$\bar{u}_4 = \ddot{\psi}_d - \lambda_4\dot{\psi} - k_\psi \text{sign}(\sigma_\psi) \quad (24)$$

The above process fully specifies the control inputs  $\bar{u}_1$ ,  $\bar{u}_2$ ,  $\bar{u}_3$  and  $\bar{u}_4$ . Using Equations (9) and (10) one recovers the original control inputs  $u = [u_1, u_2, u_3, u_4]^T$ .

The signum function design induces a chattering phenomenon that is by no means suitable in practical situations. The design functions that are commonly used in the sliding mode practice include  $\tanh(k_0\sigma)$  where  $\tanh$  denotes the hyperbolic tangent function and  $k_0$  is a high value. Particularly, recall that one has  $\lim_{k_0 \rightarrow +\infty} \tanh(k_0\sigma) = \text{sign}(\sigma)$ .

## 5. SEPARATION PRINCIPLE

In the control system literature, it is well known that a separation principle holds locally for nonlinear control systems when exponential feedback stabilizers and exponential observers are used [20]. In this section, we show that the local separation principle holds for the quadrotor helicopter when the exponential observer is introduced into the nonlinear feedback loop associated with the asymptotic stabilizing feedback control law as shown in Figure 2. The combination of an independently designed observer and state feedback controller assures stability in the output tracking problem, where the controller and observer gains are independently designed.

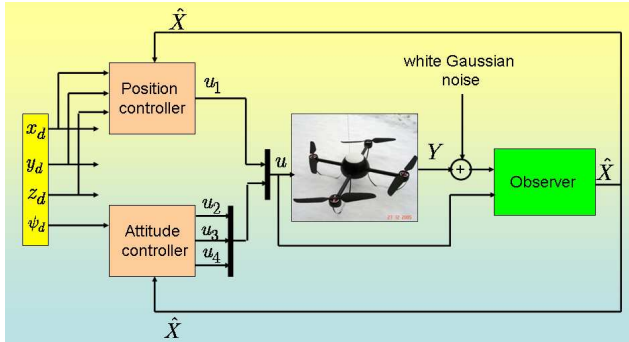


Fig. 2. Observer based-control architecture.

Consider the transformed quadrotor model by means of the local diffeomorphism  $\mathcal{Z} = \Phi(X)$  where the feedback control law is taken as function of the estimated states instead of the true ones " $u = u(\hat{\mathcal{Z}})$ ". We obtain the global closed loop of the augmented system in  $\mathcal{Z}$ -coordinates:

$$\begin{cases} \dot{\mathcal{Z}} = A(u)\mathcal{Z} + \varphi(\mathcal{Z}, u) \\ \dot{\hat{\mathcal{Z}}} = A(u)\hat{\mathcal{Z}} + \varphi(\hat{\mathcal{Z}}, u) - S^{-1}C^T(C\hat{\mathcal{Z}} - Y) \end{cases} \quad (25)$$

Letting  $e = \hat{\mathcal{Z}} - \mathcal{Z}$  be the estimation error, we obtain:

$$\begin{cases} \dot{\mathcal{Z}} = A(u)\mathcal{Z} + \varphi(\mathcal{Z}, u) - S^{-1}C^T C e \\ \dot{e} = (A(u) - S^{-1}C^T C)e + \varphi(\hat{\mathcal{Z}}, u) - \varphi(\mathcal{Z} - e, u) \end{cases} \quad (26)$$

We introduce the new tracking errors between the state estimate value and the desired one that are expressed in the new coordinates:

$\hat{e}_\xi = \hat{\xi} - \xi_d$  and  $\hat{e}_\psi = \hat{\psi} - \psi_d$ , where:

$$\hat{\xi} = [\hat{\mathcal{Z}}_{11} \ \hat{\mathcal{Z}}_{21} \ \hat{\mathcal{Z}}_{31}]^T, \quad \xi_d = [\mathcal{Z}_{11d} \ \mathcal{Z}_{21d} \ \mathcal{Z}_{31d}]^T,$$

$$\hat{\psi} = \hat{\mathcal{Z}}_{41} \text{ and } \psi_d = \mathcal{Z}_{41d}.$$

The choice of the sliding surfaces is based upon the synthesized tracking errors:

$$\hat{\sigma}_\xi = \ddot{\hat{e}}_\xi + \lambda_3\dot{\hat{e}}_\xi + \lambda_2\hat{e}_\xi + \lambda_1\hat{e}_\xi \quad (27)$$

$$\hat{\sigma}_\psi = \dot{\hat{e}}_\psi + \lambda_4\hat{e}_\psi \quad (28)$$

Taking into account that the control law  $u$  can be rewritten as  $u = u(\hat{\mathcal{Z}})$  where the actual states are replaced by their estimates. By the same argument as in Section IV, the closed loop system becomes:

$$\begin{cases} \dot{\hat{\sigma}} = -k \text{sign}(\hat{\sigma}) - S^{-1}C^T C e \\ \dot{e} = (A(u) - S^{-1}C^T C)e + \varphi(\hat{\mathcal{Z}}, u) - \varphi(\mathcal{Z} - e, u) \end{cases} \quad (29)$$

where  $\hat{\sigma} = [\hat{\sigma}_\xi \ \hat{\sigma}_\psi]^T$ ,  $k = \text{diag}(k_\xi I_3, k_\psi)$  is a block diagonal positive definite matrix and  $I_3 \in \mathbb{R}^{3 \times 3}$  is the identity matrix. Therefore, we can formulate the following result.

**Theorem 1:** *If the admissible control input  $u$  is bounded, the augmented system (29) under assumption 1 is locally asymptotically stable.*

**Proof:** To analyze closed-loop stability, we introduce the Lyapunov function:

$$V(e, \hat{\sigma}) = V_1(e) + V_2(\hat{\sigma}) \quad (30)$$

where  $V_1(e) = e^T S e$  and  $V_2(\hat{\sigma}) = \frac{1}{2} \hat{\sigma}^T \hat{\sigma}$

Let  $e = \hat{\mathcal{Z}} - \mathcal{Z}$  be the estimation error. It follows that the error dynamics can be written as:

$$\dot{e} = (A(u) - S^{-1}C^T C)e + \delta\varphi \quad (31)$$

where  $\delta\varphi = \varphi(\hat{\mathcal{Z}}, u) - \varphi(\mathcal{Z} - e, u)$

Differentiating  $V_1$ :

$$\dot{V}_1 = \dot{e}^T S e + e^T S \dot{e} + e^T \dot{S} e \quad (32)$$

we have:

$$\dot{V}_1 = e^T [\dot{S} + A^T(u)S + SA(u) - 2C^T C]e + 2e^T S \delta\varphi \quad (33)$$

where  $S$  is the solution of the differential Lyapunov equation (8). This implies:

$$\dot{V}_1 = -\Theta e^T S e - (Ce)^2 + 2e^T S \delta\varphi \quad (34)$$

Using the Schwartz inequality, we obtain:

$$\dot{V}_1 \leq -\Theta V_1 + 2\|S\|\|\delta\varphi\|\|e\| \quad (35)$$

Now, using the fact that  $\varphi$  is locally Lipschitz, we obtain:

$$\|\delta\varphi\| = \|\varphi(\hat{Z}, u(\hat{Z})) - \varphi(\hat{Z} - e, u(\hat{Z}))\| \leq \mu_\varphi \|e\|$$

Then:

$$\dot{V}_1 \leq -\Theta V_1 + 2\mu_\varphi \|S\|\|e\|^2 \quad (36)$$

For a bounded control  $u(t)$ , the matrix  $S(t)$  is bounded.

Hence, there exists a positive constant  $\rho_1$  such that for all  $t \geq 0$ , we have  $\|S(t)\| \leq \rho_1$ .

Using the fact that:  $\lambda_{\min}(S)\|e\|^2 \leq V_1 \leq \lambda_{\max}(S)\|e\|^2$ , where  $\lambda_{\min}$  and  $\lambda_{\max}$  are respectively the minimum and maximum eigenvalue of  $S$ , one has:

$$\dot{V}_1 \leq -\left(\Theta - \frac{2\mu_\varphi\rho_1}{\lambda_{\min}(S)}\right)V_1 \quad (37)$$

Choosing  $\Theta$  to achieve:  $\gamma_\Theta = \Theta - \frac{2\mu_\varphi\rho_1}{\lambda_{\min}(S)} > 0$ , we

obtain:

$$V_1(t) \leq \exp(-\gamma_\Theta t)V_1(0) \quad (38)$$

Otherwise,

$$\|e(t)\| \leq b \exp\left(-\frac{\gamma_\Theta}{2}t\right)\|e(0)\| \quad (39)$$

where  $b = \sqrt{\frac{\lambda_{\max}(S)}{\lambda_{\min}(S)}}$ . This implies that the estimation

error converges exponentially to zero.

Differentiating  $V_2$ :

$$\begin{aligned} \dot{V}_2 &= \hat{\sigma}^T \dot{\hat{\sigma}} = \hat{\sigma}^T (-ksign(\hat{\sigma}) - S^{-1}C^T C e) \\ &= -\hat{\sigma}^T ksign(\hat{\sigma}) - \hat{\sigma}^T S^{-1}C^T C e \end{aligned} \quad (40)$$

Hence, using the Schwartz inequality, we have:

$$\dot{V}_2 \leq -k_1 \|\hat{\sigma}\| + \|S^{-1}\| \|C\|^2 \|\hat{\sigma}\| \|e\| \quad (41)$$

where  $k_1 = \min\{k_\xi, k_\psi\}$ . Using some techniques regarding the Ricatti equations [17], we can show that the matrix  $S^{-1}(t)$  is also bounded. Then  $\exists \rho_2 > 0, \forall t \geq 0, \|S^{-1}(t)\| \leq \rho_2$ .

Therefore:

$$\dot{V}_2 \leq -k_1 \|\hat{\sigma}\| + \rho_2 \|C\|^2 b \exp\left(-\frac{\gamma_\Theta}{2}t\right)\|e(0)\| \|\hat{\sigma}\| \quad (42)$$

Now, for all  $t \geq 0$ , we have  $\|e(t)\| \leq b\|e(0)\|$ . It follows that:

$$\dot{V}_2 \leq -[k_1 - \rho_2 b \|C\|^2 \|e(0)\|] \|\hat{\sigma}\| \quad (43)$$

Choose  $k_1$  such that  $G = k_1 - \rho_2 b \|C\|^2 \|e(0)\| > 0$ . Hence:

$$\dot{V}_2 \leq -G \|\hat{\sigma}\| \quad (44)$$

This implies that the sliding surfaces  $\hat{\sigma} = [\hat{\sigma}_\xi \hat{\sigma}_\psi]^T$  converge asymptotically to zero in finite time. Thus the tracking errors  $\hat{\epsilon}_\xi = \hat{\xi} - \xi_d$  and  $\hat{\epsilon}_\psi = \hat{\psi} - \psi_d$  converge asymptotically to zero as  $t \rightarrow +\infty$ .

## 6. SIMULATION RESULTS

In order to evaluate the investigated control law combined with the observer design, a simulation has been performed on the complete closed loop system. The flight dynamics of the rotorcraft have been implemented in *Matlab* environment and the measured outputs have been disturbed. The measurements are used by the observer to estimate the roll and pitch angles, and all velocities. Then, the control law is computed using the observed states.

In order to apply the observer design, let us check assumption 1. Using the fact that  $\partial\varphi/\partial\mathcal{Z}$  is uniformly bounded for a bounded domain of flight. It follows that the function  $\varphi$  is local Lipschitz w.r.t.  $\mathcal{Z}$ .

The physical parameters used for the dynamic model of the quadrotor helicopter are:  $g = 9.81$ ,  $m = 0.56$ ,  $d = 0.21$ ,  $I_x = I_y = 0.0142$  and  $I_z = 2I_x$ .

After a short period of fine tuning by trial and error, we have fixed the values of the controller and observer parameters. The gain parameters of the control are:  $k_\xi = k_\psi = 10$ , the parameter values of  $(\lambda_1, \lambda_2, \lambda_3)$  and  $\lambda_4$  represent the coefficients of the polynomials  $(p+2)^3$  and  $(p+2)$ , respectively. The gain parameter of the observer is tuned at  $\Theta = 2$ .

The initial condition for the quadrotor model and local exponential observer are respectively:

$$X_0 = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T \text{ and}$$

$$\hat{X}_0 = [0.2 \ 0.2 \ 0.2 \ 0.2 \ 0.2 \ 0.2 \ 0.1 \ 0.1 \ 0.4 \ 0.2 \ 0.2 \ 0.5]^T$$

The desired position and yaw angle are chosen to be:

$$\xi_d = [2 \ 2 \ 3]^T, \psi_d = \pi/4$$

The measured inertial coordinates and yaw angle are overlaid with an additive Gaussian noise (mean value 0, amplitude value 0.2).

From Figures 3 and 4, it is clear that the actual states converge to their estimates and the estimate states converge to their desired ones. It is concluded from the simulations that the observer and controller give satisfactory results, and the behavior of the complete closed loop system is stable. The results of estimation errors given in Figures 5 and 6 show the efficiency of the observer, which clearly shows the quick convergence to zero of the estimation error. We remark the good performances and robustness of the mixed observer-controller in the presence of noisy measurements. The control signals produced to observe these results are presented in Figure 7. Figure 8 shows the sliding surfaces. It can be seen that the sliding surfaces converge to zero.

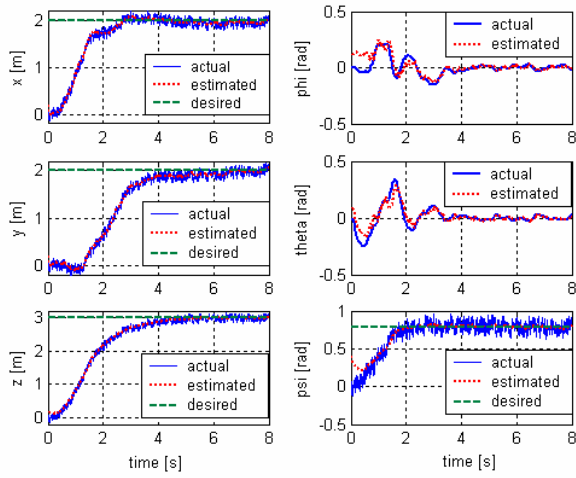


Fig. 3. Actual position and orientation and their estimates.

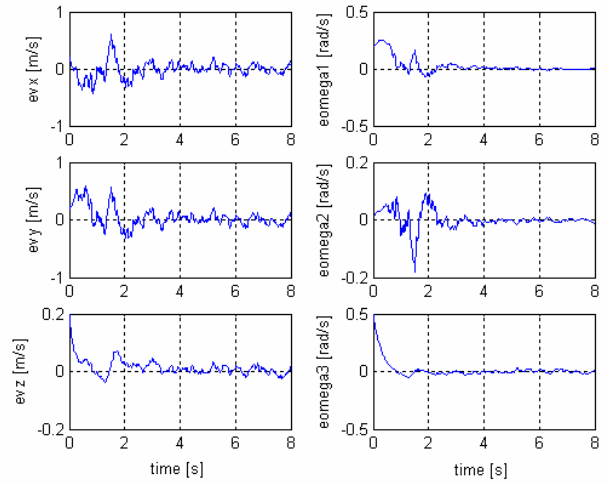


Fig. 6. Estimation error according to velocities.

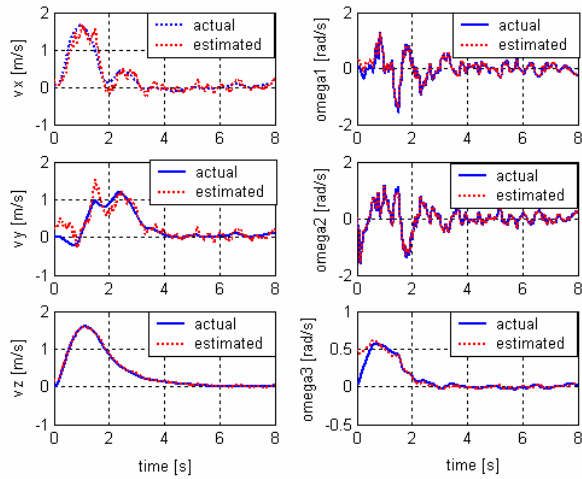


Fig. 4. Actual velocities and their estimates.

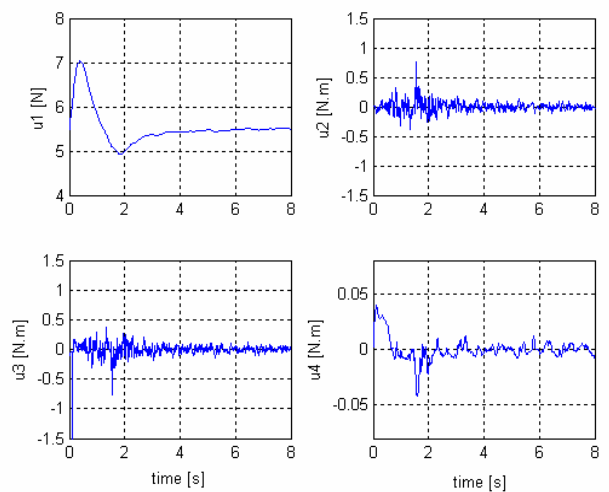


Fig. 7. Control inputs.

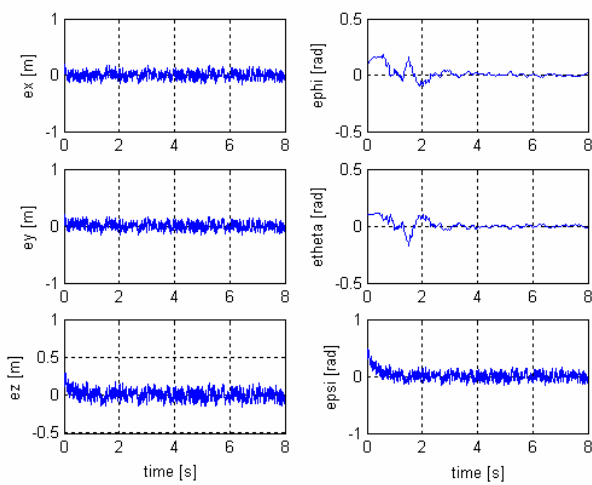


Fig. 5. Estimation error according to position and orientation.

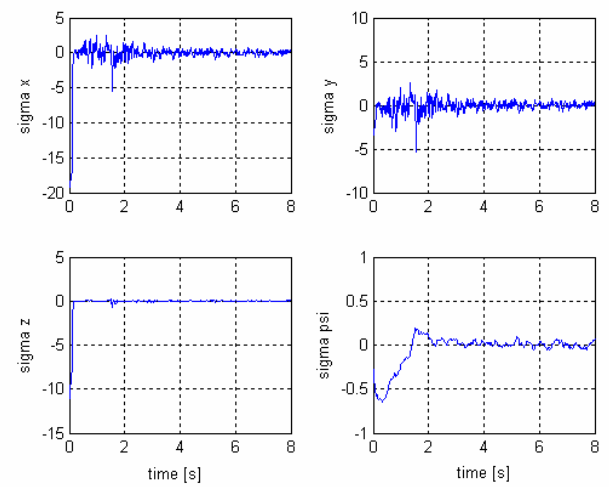


Fig. 8. Sliding surfaces.

## 7. CONCLUSIONS

In this paper, we have proposed a dynamic feedback controller using a state observer for a quadrotor helicopter. The proposed solution is based on a combination of a high gain observer and a dynamic feedback sliding mode controller. Sufficient conditions have been stated such that a kind of separation principle holds. We have shown the asymptotic stability of the global closed loop system using Lyapunov analysis. The unmeasured states have been successfully reconstructed through the observer even when the output measurements are noisy. Simulation results show the effectiveness of the proposed observer-based control. In future work, we will implement the control law on a real quadrotor helicopter.

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**M'hammed Guisser** received the sciences degree in automatic in 2003 and Ph.D. degrees in engineering sciences in 2009 from Ecole Nationale Supérieure d'Electricité et de Mécanique (ENSEM), Université Hassan II, Casablanca, Morocco.

His research interests include nonlinear control theory, state observer and digital signal processing. He is involved in applications of these techniques to the control of unmanned aerial vehicles.



**Hicham Medromi** received the PhD in engineering science from the Sophia Antipolis University in 1996, Nice, France. He is responsible for the system architecture team of the ENSEM Hassan II University, Casablanca, Morocco.

His actual main research interest concern Control Architecture of Mobile Systems Based on Multi Agents Systems. Since 2003 he is a full professor for automatic and computer sciences at the ENSEM, Hassan II University, Casablanca.