

# Modeling and Analysis of Control System for a Multi-Robotic System

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*Abstract* - System automation has been an active area of research for decades. In this respect dealing with an automation system involves a number of items across various subsystems, technical disciplines, and activities. In general, two types of approaches are applied to rescue the water-based victims: air-borne vehicle (helicopter/sea-plane), and marine vehicle. Air-borne vehicle is too expensive to perform rescue operations in the developing countries. Therefore, marine vessel/river-craft shows better efficacy in terms of economy in such countries. As a result, modeling and control of a Multiple Mobile Robots System (MMRS) has been presented [1,3]. However, an advanced control analysis has not been carried out in the MMRS. Its control system modeling and analysis is presented in this paper. The transfer functions, on behalf of track keeping and mechanical analysis of the robotic boat system, are analyzed and established. Algorithms have been developed by observing the dynamic behavior of each rescue boat.

*Index Terms* – Autonomous Robotic Boat

## ILLUSTRATIVE SYMBOLS

$M$  – Mass of the the boat including stator of the motor,  
 $J_1$  – Inertia of the rotor of the motor,  
 $J_2$  – Inertia of the propeller,  
 $D_2$  – Damping between rotor and stators,  
 $D_L$  – Damping between propeller and water surface.  
 $\psi$  – Heading angle,  
 $k$  – Rudder gain,  
 $\delta$  – Radar angle,  
 $N_1$  – Teeth of rotor gear of the motor,  
 $N_2$  – Teeth of propeller gear,  
 $u, u'$  – Surge speed and acceleration,  
 $v, v'$  – Sway speed and acceleration,  
 $r, r'$  – Yaw speed and acceleration,  
 $I_z$  – Moment of inertia with respect to Z axis,  
 $T$  – Yaw mode time constant,  
 $I_a$  – Armature current,  
 $R_a$  – Armature resistance,  
 $V_b$  – Back emf,  
 $E_a$  – Applied voltage.

This research was supported by the IUT, and partly by M.M. Ali.

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## 1. INTRODUCTION

System automation has been an active area of research for decades. In this respect dealing with an automation system involves a number of items across various subsystems and technical disciplines. The concurrency of these variables, activities must be followed in a systemic structure of a system. To deal with so many issues within a limited time, the entire rescue system activities are distributed among the teams consisting of members with expertise in the relevant fields. The activity involves the coordination and communication, and the associated data [2,4,5,6,7].

An air-borne vehicle is too expensive to perform rescue operation in some developing countries. Therefore, marine vessel shows better efficacy and convenience in the view of economy of such countries. Modeling and control of Multiple Mobile Robots System (MMRS) are presented previously [1,3,7] where an advanced study on control analysis has not been carried out. Hence, it is an issue of importance to perform an analysis of a robotic boat by which a rescue operation can be done. In a group-robotic rescue, the most important is that the rescue boats must perform a rescue operation with consistent stability [1,3,4,5]. The rescue boats should be operated from a Base Station (BS). The nearest base station from the risk ship helps the latter at first [1].

The experiments have been worked out sequentially by managing the control and the flow of the data as smoothly as possible in a systematic manner. Hence, the organization of this paper is as follows: Section 2 describes related works, Section 3 discusses the models, Section 4 focuses on experimental results, and Section 5 concludes the paper.

## 2. RELATED WORKS

A number of researchers [1,3,4,5] have referred to a water based robotic system. There are several research projects in the field of autonomous robotic boat. Sarker and Hussain [1] present a simple prototype of sensing and control of a water based robotic system where initial works on RB relationship, velocity, computer environment and so forth are discussed. Few sensory circuits along with their simulated sensor data have been presented. Leghari *et al.* [3] propose a model of a multi-

mobile robot system where preliminary principles and methods towards an automated system are outlined. Dhariwal and Sukhatme [4] present an algorithm for estimating robotic boat location by integrating various sensor inputs. Multi-sensors are adopted in this system. The previous works fall short of attention to the control analysis of group-robotic boats. To find a solution, the following steps are taken:

- To design mechanical and electromechanical models;
- To find out transfer functions - electrical and mechanical ;
- To develop algorithms for simulation and;
- To analyze controllability and stability.

### 3. THE SYSTEM

This section deals with organization, presentation and analysis of different models of an autonomous robotic boat, and draws valid conclusions and thereafter makes reasonable decisions on the basis of analysis results. Each RB is 1 foot long, 0.5 foot width at the back-end, and 0.27 foot width at the middle front-end in the lab. The three boats are placed in a 20'x30' pond as shown in the Fig.1. The RB-to-RB communication setup is developed with networking, and an RB is linked with a computer and each computer communicates with one another.

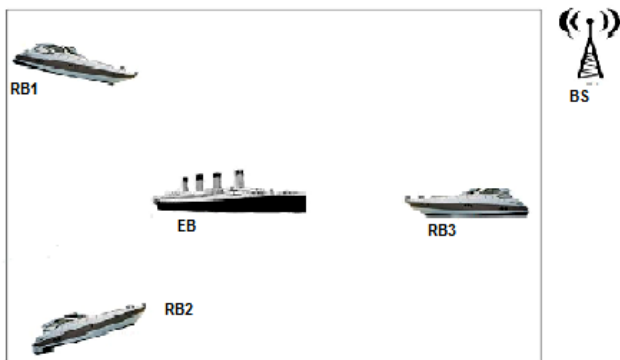


Fig. 1: The overall system

#### 3.1 Computer Environment

We use WINDOWS workstations that are connected with Ethernet LAN for an experimental implementation. The multi-tasking functionality and a foreign-function

interface is done with the help of a base programming language where Common Lisp Object System (CLOS) is exists. The multi-tasking functionality can fork sub-processes, wait for a termination of sub-processes and lock the other process execution. This is necessary for establishing parallel operation within an RB, and for enabling to receive and to handle data coming from other RB. MATLAB 7.0 and C language are used in this work.

#### 3.2 Motion Analysis

The concept of *Degrees of Freedom* (DoF) in the work is central to the principle of estimating statistics of an RB over water surface. There are six DoF, for example - Moving up and down (heaving), Moving left and right (swaying), Moving forward and backward (surge), Tilting forward and backward (pitch), Turning left and right (yaw), and Tilting side to side (roll) [1,4,6,7] as shown in Fig..2.

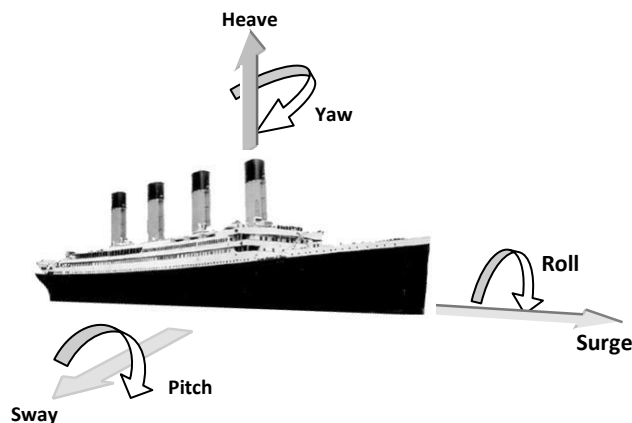


Fig. 2: Illustration of an RB with DOF

We adopt the following notations [4] by analyzing Fig.2 on behalf of the coordinate system and associated nomenclature :

Position vector in an Earth-fixed frame  $\eta_1 = [\varphi, \theta, \psi]^T$

Surge, sway and heave velocity vectors  $v_1 = [u, V, w]^T$

Roll, pitch and yaw velocity vectors  $v_l = [p, q, r]^T$

Eventually, three DOF are practically important. These lie in the plane parallel to the surface of the water, namely surge, sway and yaw. It is based upon the fact that the boat only moves in a plane parallel to the surface of water (will not go above or below water (z-axis)) and turn only along the z axis (without tilt/tip over).

The Force and Moment Equation can be presented as [1,4]:

$$\text{Surge Motion} \quad A = M(u' - rv) \dots \dots (1)$$

$$\text{Sway Motion} \quad B = M(v' + ru) \dots \dots (2)$$

$$\text{Yaw Motion} \quad N = I_z r' \dots \dots (3)$$

For a constant speed straight line motion condition, linearization of (1) to (3) decouples the surge equation. Taking the Laplace transform of the coupled sway-yaw system and cancellation of the sway term, the following 2nd order Nomoto model is obtained [4,6,7].

$$\frac{r(s)}{\delta(s)} = \frac{k(1 + T_3s)}{(1 + T_1s)(1 + T_2s)} \dots \dots \dots (4)$$

The Nomoto models that are derived under the assumption of constant speed can be used to describe the steering behavior for small rudder angles, when the loss of speed is negligible, and to describe the behavior during the stationary part of the zigzag movement, where the speed remains constant as well. However, the parameters of the models are different for different rudder angles.

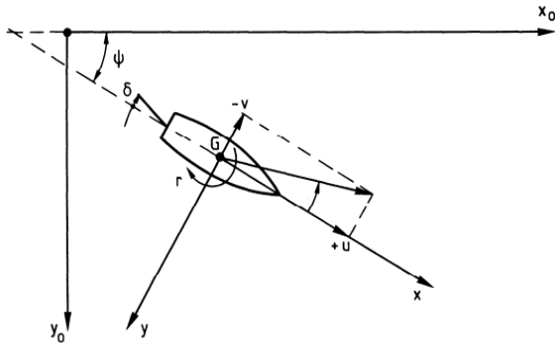


Fig. 3: General formation control configuration

According to pond-in-lab trial data-based identification results, the values of the parameters  $T_2$  and  $T_3$  in equation (4) are almost same. This suggests further simplification of (4) is possible and the 1<sup>st</sup> order Nomoto model follows:

$$\frac{r(s)}{\delta(s)} = \frac{k}{(1 + Ts)} \dots \dots \dots (5)$$

Where  $T = T_1 + T_2 - T_3$ . Equation (5) can be represented by a differential equation in the time domain as:

$$v = [u, v, r]^T$$

$$\dot{r} = -\frac{1}{T}r + \frac{k}{T}\delta \dots \dots \dots (6)$$

Equation (5) can be written as the following 2<sup>nd</sup> order model with the definition of  $r = \dot{\psi}$ ,

$$\frac{\psi(s)}{\delta(s)} = \frac{k}{s(1 + Ts)} \dots \dots \dots (7)$$

The kinematics of the ship can be described according to Fig.3 as follows:

$$\dot{x} = u \cos \psi + v \sin \psi \dots \dots \dots (8)$$

$$\dot{y} = u \sin \psi - v \cos \psi \dots \dots \dots (9)$$

$$\dot{\psi} = r \dots \dots \dots (10)$$

The simplification of equations (8) to (10) can be written as [4]:

$$\dot{x} = u + v\psi$$

$$\dot{y} = u\psi - v$$

where,  $\sin \psi \approx \psi$  and  $\cos \psi \approx 1$ , if  $\psi$  is very small and  $u$  is much larger than  $v$ . In Laplace transform, the above equations are

$$x = \frac{u}{s} + \frac{v\psi}{s}$$

$$y = \frac{u\psi}{s} - \frac{v}{s}$$

Substitute the relationship defined by equation (7)

$$x = \frac{u}{s} + \frac{kv}{s^2(1 + Ts)} \delta$$

$$y = \frac{ku}{s^2(1 + Ts)} \delta - \frac{v}{s}$$

The transfer functions can be simplified by assuming  $u/s$  and  $v/s$  very small:

$$\frac{x}{\delta} = \frac{kv}{s^2(1 + Ts)}$$

$$\frac{y}{\delta} = \frac{ku}{s^2(1 + Ts)}$$

Finally, it yields to a generalized form as :

$$\frac{(x, y)}{\delta} = \frac{k(v, u)}{s^2(1 + Ts)} \dots \dots \dots (11)$$

### 3.3 Mechanical Analysis

The input-output relationship of a mechanical system can be characterized by using a transfer function. The transfer function of a system is a mathematical model in which it is an operational method of expressing the differential equation that relates the output variable to the input variable. It is a property of a system itself, independent of the magnitude and nature of the input or driving factor as shown in Fig.4 [1].

After simplification we get the following equivalent inertia ( $J_{eq}$ ), torque ( $T_1$ ) and damping ( $D_{eq}$ ),

$$J_{eq} = J_1 + J_2 \left( \frac{N_1}{N_2} \right)^2$$

$$T_1 = T_2 \left( \frac{N_1}{N_2} \right)$$

$$D_{eq} = D_L \left( \frac{N_1}{N_2} \right)^2$$

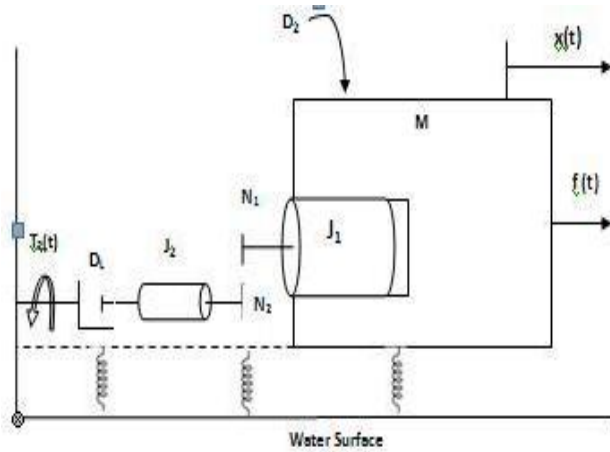


Fig. 4: Mechanical system model

The Laplace transform of the equations of motions of above mechanical system can be written as :

$$X(s)[Ms^2 + D_2s + K_{eq}] - \theta(s)[D_2s] = F(s) \dots (12)$$

$$\theta(s)[J_{eq}s^2 + (D_2 + D_{eq})s] - X(s)[D_2s] = T_1(s) \dots (13)$$

Considering that  $x_1$  is the linear distance travelled by the rotating propeller and  $f_a$  is the axial force that drives the boat in the forward direction, we find the following relations :

$$x_1 = r\theta$$

$$T_1 = rf_a \cos\theta_1$$

Replacing the values of  $\theta(s)$  and  $f(s)$  in equation (12) and (13), we have

$$X(s)[Ms^2 + D_2s + K_{eq}] - \frac{X_1(s)}{r}[D_2s] = -F_a(s)$$

$$X_1(s) \left[ \frac{J_{eq}}{Kr} s^2 + \frac{(D_2 + D_{eq})}{Kr} s \right] - X(s)[D_2s] = F_a$$

where  $K = r \cos\theta_1$

Therefore, the final transfer function can be written as below [1] where X is output of displacement, F is input of force.

$$\frac{x(s)}{F_a(s)} = \frac{\left( \frac{J_{eq}}{kr} s^2 + \frac{(D_2 + D_{eq})}{kr} s \right) + \frac{D_2s}{r}}{M \frac{J_{eq}}{kr} s^4 + D_2 \frac{J_{eq}}{kr} s^3 + \frac{k_{eq} J_{eq}}{kr} s^2 + \left( \frac{D_2 + D_{eq}}{Kr} \right) Ms^3 + \left( \frac{D_2 + D_{eq}}{Kr} \right) D_2 s^2 + \left( \frac{D_2 + D_{eq}}{Kr} \right) k_{eq} s - \frac{D_2^2}{kr} s^2} \dots (14)$$

### 3.4 Electro-Mechanical Analysis

A system consisting of electrical and mechanical variables is called electromechanical system [1,3,4]. An RB is an electromechanical component that yields a displacement output for voltage input.

From Equation (12)

$$X(s) = \frac{\theta(s)[D_2s] + F(s)}{Ms^2 + D_2s + K_{eq}} = \frac{\theta(s)[D_2s] - \frac{T_1}{r \cos\theta_1}}{Ms^2 + D_2s + K_{eq}}$$

From Equation (13)

$$\theta(s)[J_{eq}s^2 + (D_2 + D_{eq})s] - \frac{\theta(s)[D_2s]^2 - \frac{T_1}{r \cos\theta_1}[D_2s]}{Ms^2 + D_2s + K_{eq}} = T_1(s)$$

$$\theta(s) \left[ \frac{(J_{eq}s^2 + (D_2 + D_{eq})s)(Ms^2 + D_2s + K_{eq}) - (D_2s)^2}{Ms^2 + D_2s \left( 1 + \frac{1}{K} \right) + K_{eq}} \right] = T_1(s)$$

Now,

$$R_a I_a(s) + L_a s I_a(s) + V_b(s) = E_a(s)$$

$$\frac{(R_a + L_a s) T_1(s)}{K_t} + K_b s \theta(s) = E_a(s)$$

$$T_1(s) = \frac{K_t (E_a(s) - K_b s \theta(s))}{R_a}$$

Therefore,

$$\frac{\theta(s)}{E_a(s)} = \frac{1}{\frac{R_a}{K_t} \left[ \frac{(J_{eq}s^2 + (D_2 + D_{eq})s)\{(Ms^2 + D_2s + K_{eq}) - [D_2s]^2\}}{Ms^2 + D_2s \left( 1 + \frac{1}{K} \right) + K_{eq}} \right] + K_b s}$$

### 4. RESULTS AND DISCUSSIONS

The stability of the track keeping control system, mechanical system and Electro-mechanical analysis of a robotic boat are simulated and examined in this section by using the developed mathematical model.

We will analyze the system for stability using step response, impulse response, pole zero map, root locus and bode plot.

### 4.1 Tracking Control

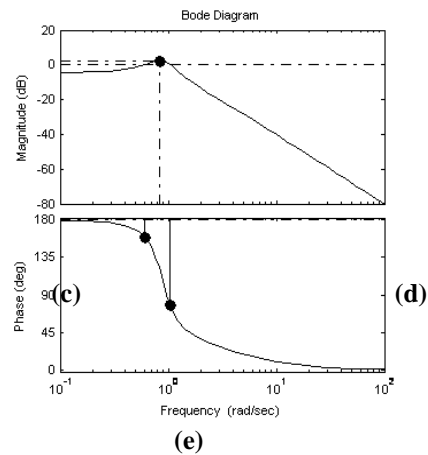
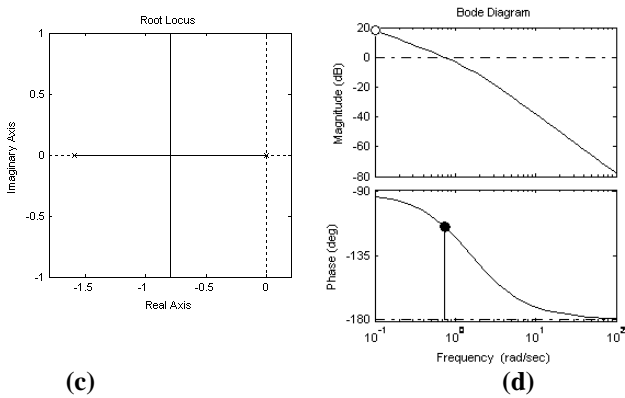
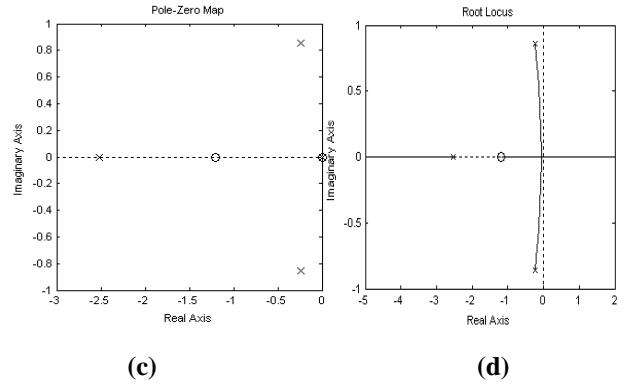
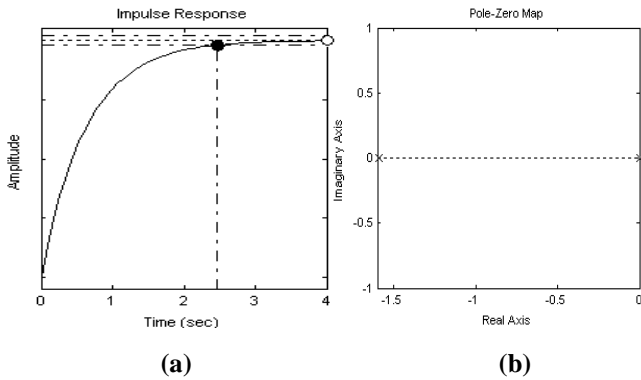


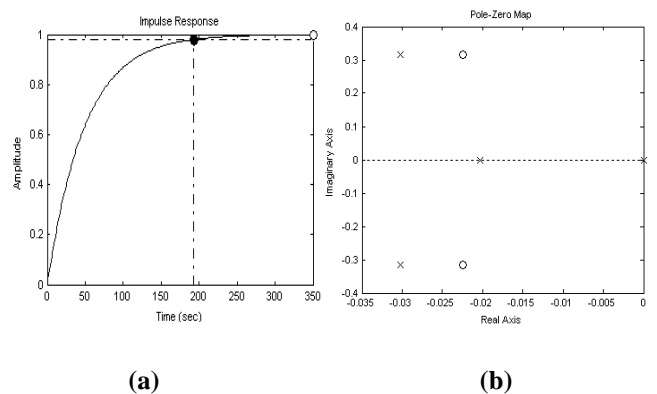
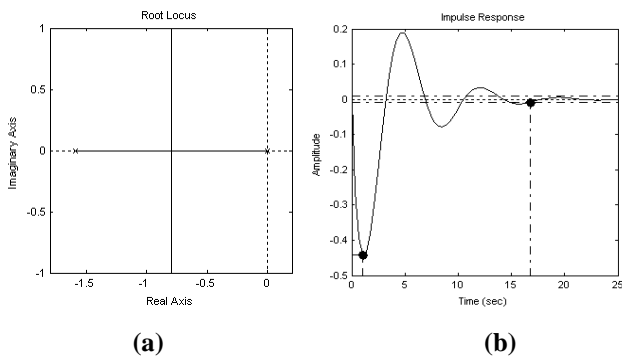
Fig. 5: Computer simulation (a) Impulse response, (b) Pole-Zero (c) Root Locus, (d) Bode plot

Fig. 6: Computer simulation (a) Step, (b) Impulse, (c)Pole-zero, (d) Root Locus , (e) Bode plot.

Steady-state analysis has been carried out to specify system performance, disturbance inputs, non-unity feedback and so forth. Root locus is used as a method for stability and transient responses. Open-loop poles and zeros, and calculation of both departure and arrival angle have been worked out. The present study has verified the current status of robotic boat in the improvement of transient and steady-state error response. According to the Nyquist criterion, a feedback system is stable if the loop gain is less than 0dB at angle 180°. Meanwhile, we can determine some margins of stability to show how close the system instability is.

The mechanical analysis of an RB, is an issue of a system with gears. Gears are present in the electric motor and provide mechanical advantage to rotational system as shown in Figure.4. Gears expose backlash, which occurs because of the loose fit between two meshed gears i.e., gears are not lossless. The rotor gear of the motor and the propeller gear are rotated at an angle 9.87° and the gear ratio is 1:10. The simple specification of an RB,  $J_1$  is 1.36 kg-m<sup>2</sup>,  $D_2$  is 0.87 N-m s/rad,  $J_2$  is 195 kg-m<sup>2</sup>, and  $M$  is 2.23 kg.

### 4.2 Mechanical Responses



### 4.3 Electro-Mechanical Responses

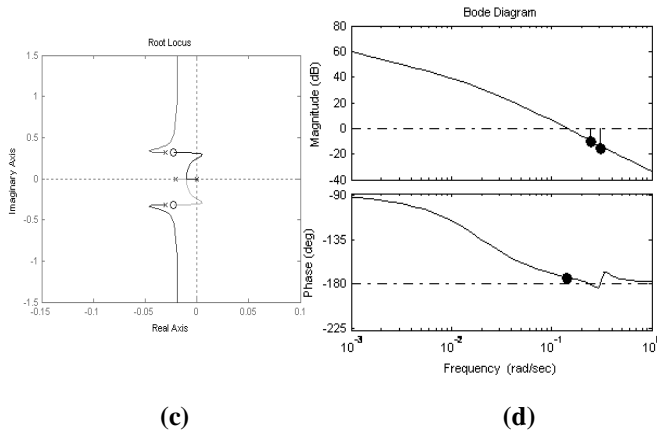


Fig. 7: Computer simulation without compensator (a) Impulse, (b) Pole-zero, (c) Root Locus, (d) Bode plot

A magnetic field is developed called ‘fixed field’ in an RB electro-mechanical study. A rotating circuit through which current  $I_a = 0.38\text{mA}$  flows, the resistance is  $R_a = 94\text{k}\Omega$ ,  $V_b$  is the back emf,  $E_a = 35\text{ Volt}$  is the applied voltage. Numerical simulations show the effectiveness of the analysis because control logics are observed and applied in it. An additional force term is added to the translational equations of motion in the numerical integration. The robotic boat successfully achieves a new desired formation.

A comparative assessment of the computation time between the proposed technique and the robotic boat localization (SAC) proposed by Dhariwal and Sukhatme [4] was carried out. This is shown in Table 1, where  $K$  represents the flow rate of sensed data as referred by [1], and a single stage change of the RB variables (velocity, motor torque etc.) towards the desired output is denoted as an *execution*. It is clear that a 16-dimensional space provides sufficient output

Table 1: Comparison of the proposed method and SAC.

	Our method K=16	Our method K=8	SAC
No of execution	302	285	365
Output	100 %	58.4 %	100 %

**5. CONCLUSIONS AND FUTURE WORKS**

The general principles of an autonomous robotic boat have been presented. The transfer functions regarding track keeping, mechanical and electro-mechanical models of a robotic boat have been analyzed and developed. Stability analysis of the mentioned models has been carried out by simulating the transfer functions using different techniques.

There are a number of major research challenges that still need to be overcome for a full potential system, such

as, fuzzy logic based automated control, fault analysis, wind effects (lateral), and design of a controller for improving stability. This is an ongoing research and it needs continuous improvement.

**ACKNOWLEDGEMENTS**

Special Thanks to IUT for continuous financial aid for Scientific Research and Development in Bangladesh (SRDB). We would like to thank A/Prof. Dr. Z.M. Hussain (Senior MIEEE), Prof. Dr. K K Islam (FIEB), Prof. Dr. Md. Saifur Rahman (FIEB) for their helpful discussions.

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