An Eff cient Non-stationary Jammer Filtering Method for Spread Spectrum Conditioning

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Abstract— In this paper, we propose a new technique to effectively suppress non stationary jamming in direct sequence spread spectrum (DS/SS) communication systems. This technique combines a new scheme of instantaneous frequency (IF) estimation with the projection based interference suppression. In order to capture jammer subspace, a time-varying autoregressive (TV-AR) modeling is used to estimate the IFs of the non stationary jammer signals. Orthogonal polynomials are used for the basis function of the TV-AR model to reduce the computational complexity. The jammer subspace is constructed from the models governed by the estimated time-varying IFs. The estimated jamming interference is removed from the received data by subspace projection, resulting in less distortion to the desired signal. The performance of this approach is analyzed and compared with approaches using time-varying notch filtering.

Index Terms—Time-varying AR model, non-stationary jammer, instantaneous frequency, subspace projection, orthogonal base function.

1. INTRODUCTION

IRECT-sequence spread-spectrum (DS/SS) communica-**D** tion systems have a certain degree of inherent immunity to intentional and/or unintentional jamming [1]. However, in some applications, the jammer might be much stronger than the desired DS/SS signal, and the processing gain due to spreading might be insufficient to provide enough jamming resistance for decoding the useful signal reliably. In such cases, jammer suppression needs to be done prior to symbol detection. For stationary interference, many jammer mitigation techniques have been developed to remove its effect adequately [2] [3]. However, the non stationary interferences cannot be adequately suppressed using a single domain mitigation algorithm due to the fact that the signal parameters are time-varying. Frequency modulated (FM) interferers are such examples. Many interference suppression techniques are reported and most of the techniques use IF estimation [4] and time-frequency analysis (TFA) [5]. The disadvantage of TFA is its large computational burden and slow convergence.

To speed up convergence and lower the computational complexity of TFA, this paper proposes to estimate the IF, based on TV-AR modeling [6]. Parametric analysis and modeling of signals using a TV-AR model with time varying coeff cients has found applications in a variety of contexts including non stationary signal processing, spectral estimation, radar signal processing and others [7] [8]. In communication it is an eff cient scheme for suppressing rapidly varying non stationary interference [8] [9].

There are several potential advantages to use time-varying AR [7] [10]. In some cases the system model may be more realistic since it allows for the continuously changing behavior of the signal. This should lead to increased accuracy in signal representation. In addition, the method may be more eff cient since the inclusion of time variations in the model should allow analysis over longer data windows. TV-AR modeling approaches are generally good for detecting multiple timevarying spectral peaks or the IF, and it can parameterize a non-stationary process with a small number of parameters. However, for the time-varying linear prediction method, the predictor coefficients are obtained by solving a set of linear equations. Because the number of the coeff cients increases linearly with the number of terms in the series expansion, there is a signif cant increase in the amount of computation for time-varying AR as compared with traditional AR. In order to reduce computational complexity, this paper investigates a TV-AR model based on orthogonal polynomial functions as the the base function [11]. Then the estimated cross-correlation matrix and auto correlation vector used to calculate the time varying coeff cients are in a much simpler format. Therefore, the computational complexity is dramatically reduced.

The f nite impulse response (FIR) notch f lter based interference suppression method [8] [12] is simple to implement once an IF estimate is determined. However, the notch f lter distorts the desired signal when it cuts the interference. The jammer IF can construct the jammer subspace. The respective orthogonal subspace projection is used to excise the jammer. It has been demonstrated recently [13] that the subspace projection method can robustly improve the output signal to noise ration (SNR) signif cantly. As the jammer subspace is solely determined by the jammer IF, reliable estimation of the IF is important for FM interference mitigation.

We then use the time-varying notch flter and orthogonal projection to suppress the jammer based on the estimated IF [14]. Analysis shows that the proposed approach of orthogonal polynomial basis vectors in the TV-AR fltering signif cantly reduces the computational complexity. Simulation results demonstrate that the proposed method outperforms other approaches using regular polynomials. Orthogonal projection can improve the bit error rate (BER) performance compared with the time-varying notch flter approach.

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Fig. 1. Signals (SNR=1dB and JRS=40dB) (a) signal after spreading. (b) jammer signals. (c) received signal (include spreading signal, noise and jammer)

2. SIGNAL MODEL

In DS/SS communications, each information symbol is spread using a length-L spreading code. That is,

$$d(k) = s(n)c(n,l) \quad \text{with} \quad k = nL + l, \qquad (1)$$

where s(n) is the symbol-rate information bearing signal, and c(n, l) is the binary spreading sequence of the *n*th symbol. We use c(n) instead of c(n, l) for simplicity when no confusion arises. The received chip rate fltered by a matched flter and sampled data sequence can be expressed as the product of the chip-rate sequence d(k) and its spatial signature h,

$$p(k) = d(k)h \tag{2}$$

Within a symbol interval, after chip-rate processing, the received data become

$$\mathbf{x} = \mathbf{p} + \mathbf{e} + \mathbf{j} \tag{3}$$

where

$$\mathbf{p} = \left[\begin{array}{cccc} p_1 & p_2 & \cdots & p_L\end{array}\right]^T$$

is an $L \times 1$ vector containing signal of interest, the white noise vector is

 $\mathbf{e} = \begin{bmatrix} e_1 & e_2 & \cdots & e_L \end{bmatrix}^T$

and the FM jammer vector with length L is

$$\mathbf{j} = \begin{bmatrix} j_1 & j_2 & \cdots & j_L \end{bmatrix}^T.$$

Each element in the vector is the received signal at time n

$$x(n) = p(n) + e(n) + j(n)$$
(4)

Figures 1 and 2 show the spreading signal, linear FM jammer and received signal in time domain and their spectrogram. One can f nd that the received signal is corrupted by the high level jammer when the jammer to signal ration (JSR) is high.



Fig. 2. Spectrum gram of signals (SNR=1dB and JSR=40dB)(a) signal after spreading. (b) jammer signals. (c) received signal (include spreading signal, noise and jammer)

3. IF ESTIMATION BASED ON TV-AR MODEL

3.1. TV-AR Modeling

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In traditional AR modeling, the discrete-time data x(n) is modeled as a linear combination of its past p samples. That is:

$$x(n) = -\sum_{i=1}^{P} a_i x(n-i) + e(n)$$
(5)

where e(n) is the modeling error/residule.

If the strong jammer contained in x(n) has non-stationary or time-varying parameters, say an FM jammer, we can use a TV-AR model [10] to improve the modeling accuracy. In such a model, the f lter coeff cients, a_i 's, are allowed to change with time. A time-varying autoregressive (TV-AR) process is then expressed as:

$$c(n) = -\sum_{i=1}^{P} a_i(n) x(n-i) + e(n)$$
(6)

where the AR coeff cients are constructed from

$$a_i(n) = \sum_{k=0}^{q} a_{ik} u_k(n),$$
(7)

where $u_k(n), (k = 0, 1, \dots, q)$ is a set of independent basis functions.

To estimate the FM signal, we choose the basis function as follows: $\{u_k(n) = n^k\}_{k=0}^q$. Other choices of basis functions leading to reduced computation are under investigation.

Combining (5) and (6), the prediction equation becomes:

$$x(n) = -\sum_{i=1}^{p} \left(\sum_{k=0}^{q} a_{ik} u_k(n) \right) x(n-i) + e(n)$$
 (8)

For coeff cient calculation, we use an optimization criterion of minimizing the total squared error, i.e.,

$$E = \sum_{n} \left(x(n) + \sum_{i=1}^{p} \sum_{k=0}^{q} a_{ik} u_k(n) x(n-i) \right)^2$$

Minimizing the above error with respect to each coeff cient leads to the following set of p(q + 1) equations:

$$\sum_{i=1}^{p} \sum_{k=0}^{q} a_{ik} c_{kl}(i,j) = -c_{0l}(0,j)$$
$$0 \le j \le p, \ 1 \le l \le q$$
(9)

where $c_{kl}(i, j)$ is termed as a generalized correlation function, defined as,

$$c_{kl}(i,j) = \sum_{n} u_k(n)u_l(n)x(n-i)x(n-j)$$
$$0 \le k, l \le q$$
(10)

Expressed in a matrix form, equation (9) becomes

$$\underbrace{\begin{bmatrix} \mathbf{C}_{00} & \mathbf{C}_{01} & \cdots & \mathbf{C}_{0q} \\ \mathbf{C}_{10} & \mathbf{C}_{11} & \cdots & \mathbf{C}_{1q} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_{q0} & \mathbf{C}_{q1} & \cdots & \mathbf{C}_{qq} \end{bmatrix}}_{\mathbf{C}} \underbrace{\begin{bmatrix} \mathbf{a}_{0} \\ \mathbf{a}_{1} \\ \vdots \\ \mathbf{a}_{q} \end{bmatrix}}_{\mathbf{a}} = -\underbrace{\begin{bmatrix} \mathbf{c}_{0} \\ \mathbf{c}_{1} \\ \vdots \\ \mathbf{c}_{q} \end{bmatrix}}_{\mathbf{c}} \quad (11)$$

where

C

$$\mathbf{a}_{i} = \begin{bmatrix} a_{1i} & a_{2i} & \cdots & a_{pi} \end{bmatrix}^{T}, \ 0 \le i \le q$$
$$\mathbf{a}_{i} = \begin{bmatrix} c_{0i}(0,1) & c_{0i}(0,2) & \cdots & c_{0i}(0,p) \end{bmatrix}^{T}$$
$$0 \le i \le q$$

and

$$\mathbf{C}_{kl} = \begin{bmatrix} c_{kl}(1,1) & c_{kl}(1,2) & \cdots & c_{kl}(1,p) \\ c_{kl}(2,1) & c_{kl}(2,2) & \cdots & c_{kl}(2,p) \\ \vdots & \vdots & \ddots & \vdots \\ c_{kl}(p,1) & c_{kl}(p,2) & \cdots & c_{kl}(p,p) \end{bmatrix}$$

is a block Toeplitz matrix.

The solution for providing the coeff cients needed in (7) is

$$\mathbf{a} = -\mathbf{C}^{-1}\mathbf{c} \tag{12}$$

3.2. TV-AR Model and Different Base Functions

For the time-varying linear prediction method outlined in Section 3.1, the predictor coeff cients are obtained by solving a set of linear equations given by (9). Because the number of the coeff cients increases linearly with the number of terms in the series expansion (q + 1), there is a signif cant increase in the amount of computation for TV-AR as compared with traditional AR (with q = 0). In TV-AR modeling, each AR coeff cient is expanded by a set of bases. This section focuses on the computational aspect based on different base functions.



Fig. 3. Different bases (a) Power of time (b) Trigonometric fuction base (c) The Chebyshev orthogonal polynomials (d) The Hermite orthogonal polynomials

3.2.1) Powers of The Time: We can approximate a wide variety of coeff cient time variations. Powers of time functions are widely used as the base function in TV-AR modeling

$$u_k(n) = n^k \tag{13}$$

This set of bases is shown in Fig. 3(a)

In our TV-AR model, we use $u_k(n) = n^k$, n = 1, 2, ..., Nand k = 1, 2, ..., q, where N is the window size and q is the order of the base functions.

3.2.2) Trigonometric Function: The base function could be trigonometric functions, as in a Fourier series [10], as shown in Fig. 3(b)

$$u_k(n) = \begin{cases} \cos(kn\omega) & k = \text{even} \\ \sin(kn\omega) & k = \text{odd} \end{cases}$$
(14)

where ω is a constant dependent upon the length of the incoming data. In order to estimate the jammer's IF, in the TV-AR model

$$u_0(n) = \cos(0n\omega)$$
(15)

$$u_1(n) = \sin(1n\omega)$$

$$u_2(n) = \cos(2n\omega)$$

where we choose $\omega = \pi/N$, n = 1, 2, ..., N, N is window size.

3.2.3) Chebyshev Orthogonal Polynomial: Using orthogonal polynomials instead of the the regular polynomial in TV-AR coeff cient f tting reduces computation signif cantly. A set of orthogonal polynomials defined as the solutions to the Chebyshev [15] differential equation and denoted $T_n(x)$, can be found in Fig. 3(c).

The Chebyshev polynomials are orthogonal polynomials with respect to the weighting function $(1 - x^2)^{-1/2}$

$$\frac{2}{\pi} \int_{-1}^{1} T_n(x) T_m(x) \frac{dx}{\sqrt{1-x^2}} = \begin{cases} 0 & (n \neq m) \\ 1 & (n = m \neq 0) \\ 2 & (n = m = 0) \end{cases}$$
(16)

For our particular TV-AR model for IF estimation, we use

$$u_0(x) = T_0(x) = 1$$
(17)

$$u_1(x) = T_1(x) = x$$

$$u_2(x) = T_2(x) = 2x^2 - 1$$

where $x = \frac{2n}{N} - 1$, n = 1, 2, ..., N and N is the total number of data in the window, and $x \in [-1, 1]$.

3.2.4) Hermite Orthogonal Polynomial: The Hermite polynomials are set of orthogonal polynomials over the domain $(-\infty, \infty)$ with weighting function e^{-x^2} [15], as shown in Fig. 3(d). They are

$$H_n(x) = (-1)^n e^{x^2} \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(-1)^k n!}{k! (n-2k)!} (2x)^{n-2k}$$
(18)

$$\int_{-\infty}^{\infty} H_m(x) H_n(x) e^{-x^2} dx = \begin{cases} 0 & (n \neq m) \\ 2^n n! \sqrt{\pi} & (n = m) \end{cases}$$
(19)

In the proposed TV-AR model, we use

$$u_0(x) = H_0(x) = 1$$

$$u_1(x) = H_1(x) = 2x$$

$$u_2(x) = H_2(x) = 4x^2 - 2$$
...
(20)

where x = n - 2/N, n = 1, 2, ..., N, N is the window size, $x \in [-\infty, \infty]$.

3.3. Calculational Complexity

Since the orthogonal polynomials are orthogonal to each other, from (12) only the sub matrix on the diagonal of the big block matrix exists, and the other sub matrix is a zero matrix. We can reduce the complexity by just calculating the sub matrix diagonal. The auto correlation matrix C becomes a block diagonal matrix when we use orthogonal polynomials as the base.

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{00} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{11} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{C}_{qq} \end{bmatrix}$$
(21)

and the cross correlation matrix c becomes:

$$\mathbf{c} = \begin{bmatrix} \mathbf{c}_0 & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}^T$$

For non orthogonal bases, the computational complexity for computing the auto correlation matric **C** is $\frac{3(q+1)^2}{2}p^2N$ multiplications and $\frac{(q+1)^2}{2}p^2N$ additions. The complexity for calculating the cross correlation vector **c** is 3p(q+1)N multiplications and p(q+1)N additions.

For orthogonal bases, the complexity for calculating the auto correlation matrix **C** is $\frac{3(q+1)}{2}p^2N$ multiplications and $\frac{(q+1)}{2}p^2N$ additions. The complexity for calculating the cross correlation vector **c** is 3(q+1)N multiplications and (q+1)N additions, as shown in Table I.

TABLE I

COMPUTING COMPLEXITY FOR COEFFICIENT MATRIX

	multiplications	additions
non orthogonal base ${\bf C}$	$\frac{3(q+1)^2}{2}p^2N$	$\frac{(q+1)^2}{2}p^2N$
non orthogonal base \mathbf{c}	3p(q+1)N	p(q+1)N
orthogonal base \mathbf{C}	$\frac{3(q+1)}{2}p^2N$	$\frac{(q+1)}{2}p^2N$
orthogonal base \mathbf{c}	3(q+1)N	(q+1)N

In order to get the time-varying coefficient, we should calculate the inverse of C.

$$\mathbf{C}^{-1} = \begin{bmatrix} \mathbf{C}_{00}^{-1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{11}^{-1} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{C}_{qq}^{-1} \end{bmatrix}$$

Therefore, we change from calculating $(q + 1)p \times (q + 1)p$ matrix inverse into q + 1 small matrix $(p \times p)$ inverse.

3.4. IF Estimation

It is well known that in the stationary case narrowband signals embeded in white noise can be modeled as an AR process and we can achieve high-resolution frequency estimation through such modeling. Similarly, for nonstationary jammer components, high-resolution IF estimation can be achieved through a TV-AR modeling approach.

For a complex-valued signal consisting of M FM components in white noise, we model the signal with a TV-AR model with order p = M. The additional modeling parameter q in (7) is selected according to priori knowledge of the signal.

From the estimated \hat{a}_{ik} , we can construct the TV-AR coeff cients $\hat{a}_i(n)$ from (7). And the time-varying transfer function corresponding to the TV-AR model can be expressed as

$$H(z,n) = \frac{1}{1 + \sum_{i=1}^{p} \hat{a}_i(n) z^{-i}}$$
(22)

By rooting the polynomial formed by TV-AR linear prediction f lter $(1+\sum_{i=1}^{p} \hat{a}_i(n)z^{-i})$ at each instant n, we can get the time-varying poles: $z_i(n)$, $i = 1, 2, \dots, p$. The instantaneous angles of certain poles provide estimate of the instantaneous frequencies f(n):

$$f(n) = \frac{ang\{z_i(n)\}}{2\pi} \quad \text{for} \quad |z_i(n)| \approx 1$$

Note that zero-padded FFTs can be used to f nd the IFs in a computationally efficient way [16].

4. FM JAMMER SUPPRESSION

Once the IFs of the FM jammer signals are estimated, we can suppress the jammer signals before de-spreading. The received data are stored in a buffer, and IF is estimated and used to remove the jammer, as shown in Fig. 4.



Fig. 4. Block diagram of the DSSS receiver with IF estimation based FM interference suppression

4.1. Subspace Orthogonal Projection

If we treat the DS/SS signal as approximately white, the received data x(n) can be viewed as FMs in white noise and modeled with a TV-AR model. We rewrite (4)

$$x(n) = j(n) + n(n) \tag{23}$$

where n(n) = e(n) + p(n) is the summation of noise and spreading signals.

The IF of the FM jammer is estimated by using the approach outlined in Section 3.

We can obtain the instantaneous phase

$$\phi(t) = \int_{-\infty}^{t} f(\tau) d\tau$$

and the discreet instantaneous phase

$$\phi(n) = \sum_{i=-\infty}^{n} f(i) = \sum_{i=0}^{n} f(i) + \phi_0$$
(24)

where ϕ_0 is the initial phase.

We represent complex-valued FM jammer mode as:

$$\mathbf{u}^T = \frac{1}{\sqrt{L}} \left[e^{j\phi(1)} e^{j\phi(2)} \cdots e^{j\phi(L)} \right]$$

where normalization by \sqrt{L} ensures unit energy.

The orthogonal projection matrix is given by [17]

$$\mathbf{P}_{\mathbf{u}}^{\perp} = \mathbf{I} - \mathbf{u}\mathbf{u}^{H} \tag{25}$$

where the vector \mathbf{u} is the unit norm base vector in the direction of the jammer vector, and the superscripts H denotes vector or matrix Hermitian [18]. Using the concept of subspace f ltering, the jammer can be removed through the projection, as shown in Fig. 5. The projection of the received signal on the orthogonal subspace of the jammer yields

$$\mathbf{x}_{\perp} = \mathbf{P}_{\mathbf{u}}^{\perp} \mathbf{x} = \mathbf{P}_{\mathbf{u}}^{\perp} \mathbf{p} + \mathbf{P}_{\mathbf{u}}^{\perp} \mathbf{e}$$
(26)



Fig. 5. Jammer excision by subspace orthogonal projection

4.2. Subspace Oblique Projection

The signal-of-interest vector **p** can be written as

$$\mathbf{p} = \begin{bmatrix} p(k) & p(k-1) & \cdots & p(k-L+1) \end{bmatrix}^T \stackrel{\Delta}{=} s(n)\mathbf{q}$$
(27)

where \mathbf{q} is the extension of the DS/SS code by replicating it with weights defined by the signal spatial signature.

$$\mathbf{q} = \begin{pmatrix} c(L-1) & c(L-2) & \cdots & c(0) \end{pmatrix}^T \otimes h \stackrel{\Delta}{=} \mathbf{c} \otimes h$$

where \otimes is the Kronecker product.

After performing the despreading matched f ltering, the symbol rate decision variable is obtained

$$y(n) = \mathbf{q}^{H} \mathbf{x}_{\perp} = s(n) \mathbf{q}^{H} \mathbf{P}_{\mathbf{u}}^{\perp} \mathbf{q} + \mathbf{q}^{H} \mathbf{P}_{\mathbf{u}}^{\perp} \mathbf{e} \stackrel{\Delta}{=} y_{1}(n) + y_{2}(n)$$
(28)

where $y_1(n)$ is the contribution of the desired DS/SS signal to the decision variable, and $y_2(n)$ is the respective contribution from the noise.

The oblique projection [19], instead of the orthogonal subspace projection matrix $\mathbf{P}_{\mathbf{u}}^{\perp}$, employs the following projection matrix

$$\mathbf{P}_{2} = \mathbf{q} \left(\mathbf{q}^{H} \mathbf{P}_{\mathbf{u}}^{\perp} \mathbf{q} \right)^{-1} \mathbf{q}^{H} \mathbf{P}_{\mathbf{u}}^{\perp}$$
(29)

From (29), since $\mathbf{P}_{\mathbf{u}}^{\perp}$ is the orthogonal to the interference, we can get that $\mathbf{P}_{2}\mathbf{q} = \mathbf{q}$ and $\mathbf{P}_{2}\mathbf{u} = \mathbf{0}$.



Fig. 6. The contour plots of the time-varying linear prediction f lters. (a) FM jammer is linear chirp. (b) The jammer is sinusoidal FM. (c) Two FM jammer signals are linear chirps. (d) Two FM jammer signals whose IFs are sinusoidal.



Fig. 7. Real IF and the estimated IF by TV-AR modeling from the synthetic signal. (a) FM jammer is linear chirp. (b) The jammer is sinusoidal FM. (c) Two FM jammer signals are linear chirps. (d) Two FM jammer signals whose IFs are sinusoidal.

Through oblique projection processing, the output of the matched flter is

5. SIMULATION RESULTS

5.1. IF Estimation and Jammer Cancelation

 $\tilde{y}(n) = \mathbf{q}^H \mathbf{P}_2 \mathbf{x} = \kappa(n) y(n)$ (30) access (CDMA) sign

Therefore, the output of the receiver using oblique projection is the orthogonal projection scaled by a scalar $\kappa(n)$.

Data containing FM jammer, an additive white Gaussian noise (AWGN) signal and a DS/SS code division multiple access (CDMA) signal are synthesized by the model in (3).

Four experiments are performed with synthesized data containing different FM signals: (a) the jammer is a linear chirp, (b) the jammer is sinusoidal FM, (c) Two jammer signals are linear chirps, and (d) Two jammers are sinusoidal FMs.

In the experiments, IF is estimated from the data using a



Fig. 8. BER vs. SNR for different jammer suppression methods (L=15, Linear FM jammer and JSR=40dB)



Fig. 9. BER vs. SNR for different jammer suppression methods (Number of jammer=2, *L*=15, Two linear FM jammer signals and JSR=40dB)

TV-AR modeling approach. Figure 6 shows the contour plots of the time-varying linear prediction flter's coeff cients from the TV-AR model. It is evident that the IF is embedded in the structure of the TV-AR coeff cients. Figure 7(a)-(d) show the real IFs and the estimated IFs using a TV-AR approach under different FM schemes.

The DS/SS signal uses Gold code of length L=15. The jammer-to-signal ratio (JSR) is 40dB. We compare the subspace projection methods and the time-varying notch flter method. Figure 8 shows the BER vs. SNR under one dimensional jammer scheme and Fig. 9 shows the BER vs. SNR under two dimensional jammer signals (2 FM jammer signals) scheme. The performance of the projection methods are much better than the time varying notch flter approach, especially with multi-jammer condition. Two kind of projections are considered. It is evident that both methods provide very close BER performance. Because the oblique projection is the orthogonal projection scaled by a scalar, it will not change the performance of the detection. Hence, it is preferred.



Fig. 10. Computation time for different of base normarize to q = 0 non-orthgonal base

5.2. Computational Complexity

Simulation results of the computational complexity are presented based on the following parameters: the order of the model p = 4, the window size N = 150, and the order of the base from 0 (the traditional AR) to 9. In simulation, we calculate the computational time of the TV-AR coeff cient estimating process (including computing the cross correlation vector, auto correlation matrix and its inverse), then we normalize the absolute time with the computation time of the original AR model using power of time as the base vector. Fig. 10 depicts the the computation time of different bases. From the simulation results, the computational complexity increases with the order q of the base. Using orthogonal polynomials as the base functions instead of the original bases can signif cantly reduce the computation amount, especially in the high base order case.

6. CONCLUSION

TV-AR modeling is an eff cient technique used in non stationary signal processing. In this paper, in order to reduce the computational complexity, the orthogonal polynomials are used as the base function in TV-AR modeling for IF estimation in DS/SS communication systems. Based on the orthogonal properties of the base vector, computational complexity of the cross correlation vector, auto correlation matrix and its inversion is signif cantly reduced. The simulation results also show the subspace projection method can provide better performance. The future research will include investigating the other projection methods and then extending the proposed method to multiple antenna scenarios.

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