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A Model Reference-Based Adaptive PID Controller for Robot Motion Control of Not Explicitly Known Systems

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Abstract - **This paper proposes a model reference tracking based adaptive PID controller with adaptive mechanisms in both feedforward and feedback paths. The objective is to force the outputs of a not explicitly known multiple input multiple output (MIMO) linear time-invariant system to track the outputs of a known reference model. A PID controller is inserted to the feedback path. The parameters of the PID controller are computed adaptively by eliminating output tracking errors. This approach allows us to manipulate the multiple motions of a complicated, unstable, or high-order robot by operating on a simpler, stable, or lower-order known reference model. The mathematic description of the robot is not required. Output matching and tracking conditions are derived and analyzed.**

Index Terms – **model reference adaptive control, PID control, output feedback, output tracking, MIMO systems.**

1. INTRODUCTION

Robust robot motion control of not explicitly known systems is a challenging subject in both military and commercial applications such as unmanned aerial vehicles (UAVs), bomb disposal mechanics, remotely-operated weapons, and satellites where the faithful mathematic description of robots is not available. Recently, many adaptive PID methods have been developed [1-12] including Back-Stepping (BS) based adaptive proportional-integral-derivative controller (PID) control which adds the integral action and nonlinear damping term to the basic back-stepping algorithm to guarantee robustness and bounded errors [1, 2], Generalized Predictive Control (GPC) based one that uses a PID controller to be equivalent to a GPC controller and incorporates the advantages of both PID and GPC [3], Heuristic Rule (HR) based one that employs rule-based switching and adaptive PID controller to perform autonomous functions [6], and many neural network based PID controllers used in various applications [7-9]. Robot motion control based on adaptive PID control has also been studied by applying an adaptive PID feedback control schemes and a feedforward input learning scheme to learn robot motions [4, 5].

The direct model reference adaptive control of MIMO systems [13, 14] uses an adaptive control structure that consists of command inputs, reference model states, and the feedback of the output errors. In an ideal situation, the outputs of the system track the outputs of the known reference model. Asymptotic stability of this algorithm is guaranteed for both the class of almost strictly positive real (ASPR) systems and the class of non-ASPR systems [15, 16] with supplementary dynamics. The robustness of system parameter variations is discussed for ASPR systems with a feedforward compensator [20, 21] and for parabolic and hyperbolic systems [22]. The non-ASPR direct model reference adaptive control algorithms have also been extended to discrete-time cases [17], and systems with unknown nonlinear functions [18]. The direct model reference adaptive control method has been applied to many practical problems [19] such as large flexible structures, robotic manipulators, drug infusion, and aircraft.

In this paper, a direct model reference output tracking (DMROT) based adaptive PID controller is proposed using both feedforward and feedback adaptive mechanisms to stabilize the closed-loop system and, at the same time, force the outputs of a multi-input multi-output (MIMO) robot system to track the outputs of a known reference model. The concept of the direct model reference adaptive control of a linear MIMO system is applied by deriving new sufficient conditions for perfect output matching and asymptotic output tracking. The paper is arranged as follows: Firstly it shows the conditions under which the ideal states exist such that the system outputs match the desired trajectories provided by a reference model. Secondly it discusses the conditions under which an ideal control raw exists so that the system states track the ideal states. Then it shows that the output tracking can be implemented using a PID controller with an adaptive mechanism. Finally, the sufficient conditions of stability are discussed and an example is presented.

2. FORMULATION OF THE SYSTEM

The proposed approach controls the motions of both stable and unstable robots to follow the ideal trajectory provided by a known reference model using a DMROT based PID controller. It allows a single controller to manipulate multiple motions of an unstable and not exactly known robot by simply driving the stable known reference model with multiple input commands as shown in Fig. 1. An adaptive PID controller is inserted into the feedback path of the DMROT structure which adds additional zeros and poles to the closed loop system to improve the stability of the robot. This PID controller also coordinates with the adaptive feedforward mechanism to control the multiple robot motions for output tracking. Since the structure of the robot is not exactly known, adaptive mechanisms are used for self-adjustment of the PID gains for achieving the best performance.

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The multi-input multi-output (MIMO) time invariant linear system with input and output disturbances is described as

$$\dot{x}_p(t) = A_p x_p(t) + B_p u_p(t) - E_p d_{ip}(t) \quad (1)$$

$$y_p(t) = C_a \bar{y}_p(t) \quad (2)$$

$$\bar{y}_p(t) = \bar{C}_p x_p(t) + d_{op}(t)$$

where $x_p(t) \in R^n$, $\bar{y}_p(t) \in R^r$, and $u_p(t) \in R^m$ are state, output, and input vectors, respectively, $d_{ip}(t) \in R^q$ and $d_{op}(t) \in R^{\bar{r}}$ are bounded input and output disturbances, and $y_p(t) \in R^{r_m}$ ($r_m \le r$) is the tracking output vector of the unknown system. Matrices A_p, B_p, C_p, and E_p are not explicitly known but their entries are assumed to be bounded and the dynamics $\{A_p, B_p, C_p\}$ is assumed to be controllable and observable. C_a is a transfer matrix which converts the output vector $\bar{y}_p(t)$ to tracking output vector $y_p(t)$. When C_a is a unity matrix, $y_p(t) = \bar{y}_p(t)$, and when $rank(C_a) < r$, dim($y_p(t)$)<dim($\bar{y}_p(t)$).

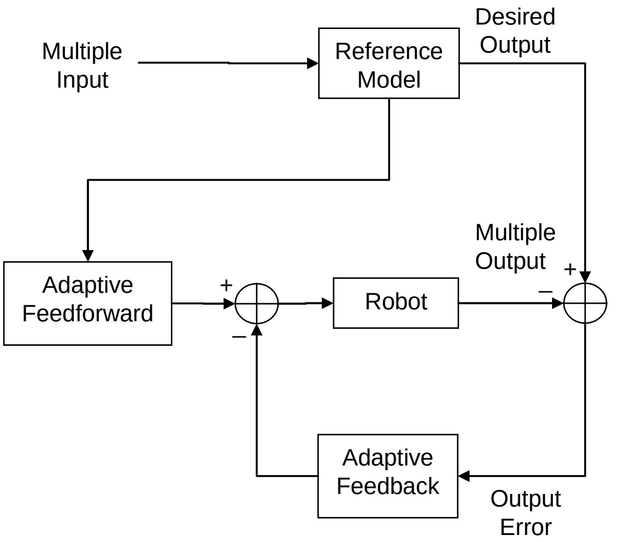

Fig. 1. Robot motion control with a reference model

The tracking output vector tracks the output of a known reference model described by a stable MIMO time-invariant system

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) \quad (3)$$

$$y_m(t) = C_m x_m(t) \quad (4)$$

where $x_m(t) \in R^{n_m}$, $y_m(t) \in R^{r_m}$, and $u_m(t) \in R^m$ are state, input, and output vectors of the reference model. Matrices A_m, B_m, and C_m are known. The output error signals, represented by vector

$$e_{yp}(t) = y_m(t) - C_a y_p(t) \quad (5)$$

are filtered by a low-pass filter

$$H_L(s) = \frac{b_f}{s - a_f} \quad (6)$$

The filtered error signals are feedback via an adaptive PID controller in order to drive the tracking output $y_p(t)$ to approach the reference model output $y_m(t)$.

The feedback mechanism is described by an adaptive PID controller as shown below

$$H_C(s) = K_P + \frac{K_I}{s} + K_D s \quad (7)$$

where the parameters K_P, K_I, and K_D are unknown and are updated adaptively. The adaptive PID controller not only provides a quick feedback response but also eliminates the tracking error between unknown system and reference model outputs. The transform function of the adaptive PID controller together with the low-pass filter is

$$H(s) = H_C(s) H_L(s) = \frac{K_D s^2 + K_P s + K_I}{s(s - a_f)} b_f \quad (8)$$

Define

$$K_D = K_{pe}, \quad (9)$$

$$K_I = K_{pf1} a_f, \text{ and} \quad (10)$$

$$K_P = -K_{pf1} - K_{pf2} - K_{pe} a_f, \quad (11)$$

to obtain the transfer function in a parallel form

$$H(s) = \frac{Z_f(s)}{E_{yp}(s)} = \left(K_{pe} - \frac{K_{pf1}}{s} - \frac{K_{pf2}}{s - a_f} \right) b_f, \quad (12)$$

where $Z_f(s)$ is the feedback signal obtained from the output of the filter $H(s)$. The state-variable representation of $H(s)$ can be described as

$$\dot{y}_{f1}(t) = e_{yp}(t) \quad (13)$$

$$\dot{y}_{f2}(t) = a_f y_{f2}(t) + e_{yp}(t) \quad (14)$$

$$z_f(t) = -K_{pf1}(t) y_{f1}(t) - K_{pf2}(t) y_{f2}(t) + K_{pe}(t) e_{yp}(t) \quad (15)$$

where $z_f(t)$ is the inverse Fourier transform of $Z_f(s)$ and $e_{yp}(t)$ is the inverse Fourier transform of $E_{yp}(s)$. Eqs. (13)-(15) can be further represented by

$$\dot{y}_f(t) = A_f y_f(t) + B_f e_{yp}(t) \quad (16)$$

$$z_f(t) = -K_{pf}(t) y_f(t) + K_{pe}(t) e_{yp}(t) \quad (17)$$

where

$$y_f(t)=\begin{bmatrix} y_{f1}(t) \\ y_{f2}(t) \end{bmatrix}\in R^{2r},\ z_f(t)\in R^m,$$

$$A_f=\begin{bmatrix} 0 & 0 \\ 0 & a_f I_r \end{bmatrix},\ B_f=\begin{bmatrix} b_f \\ b_f \end{bmatrix},\text{ and } K_{pf}=\lfloor K_{pf1} \quad K_{pf2}\rfloor.$$

The adaptive control law is described by

$$\begin{aligned} u_p(t) &= K_{pe}(t)e_{yp}(t)-K_{pf}(t)y_f(t) \\ &+K_x(t)x_m(t)+K_u(t)u_m(t) \end{aligned} \tag{18}$$

where the first and second terms are adaptive feedback signals contributed by the PID controller and the third and forth terms are adaptive feedforward signals contributed by the reference model. A block diagram is shown in Fig. 2.

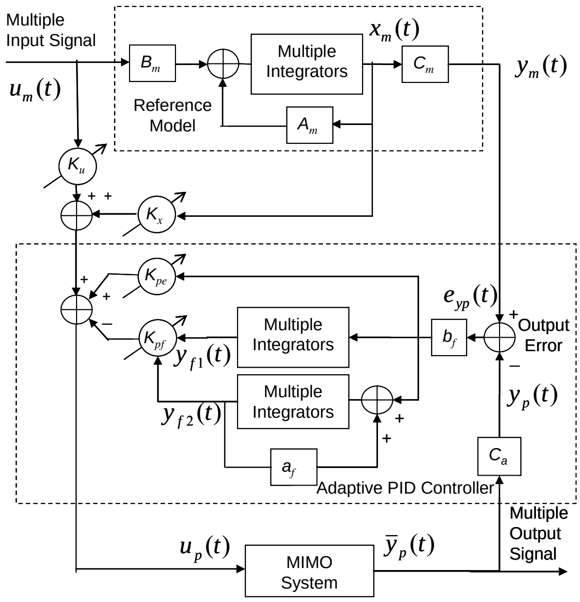

Fig. 2. Output tracking with an adaptive PID controller

A reference model $\{A_m, B_m, C_m\}$ is chosen to provide r_m desired output trajectories, represented by vector $y_m(t)$, when driven by m reference input signals, represented by vector $u_m(t)$. The not explicitly known system $\{A_p, B_p, \bar{C}_p\}$ is manipulated by m input signals, represented by vector $u_p(t)$, and generated r output signals, represented by $\bar{y}_p(t)$. In the perfect output tracking case, the r_m number of output error signals, represented by the vector $e_{yp}(t)$ approaches zero so that the tracking output $y_p(t)$ approaches the reference output $y_m(t)$. Therefore, the objective of adaptive control is to compute, without any explicit knowledge of system parameter matrices A_p, B_p and $\bar{C}_p$, the adaptive gains: K_P, K_I, *and* K_D such that the tracking output vector $y_p(t)$ of the unknown system follows the output vector $y_m(t)$ of a stable known linear reference model.

3. OUTPUT MATCHING CONDITIONS

Since the structures of system and reference model are quite different, the output of the system matches the output of the reference model only under certain conditions. In this section, we show that there exists an ideal state vector denoted by $x_p^*(t)$ such that the tracking outputs of an unknown system match the desired outputs of a reference model. Then, we demonstrate in the next section that the state vector of the unknown system $x_p(t)$ approaches the ideal state $x_p^*(t)$ with an appropriate control input applied to the unknown system. When C_a is a full rank matrix, such as unity matrix, it yields $r=r_m$. In this case, the multiple output matching is very restrictive since it requires all outputs of the unknown system track all outputs of the reference model. Mathematically

$$e_{yp}(t)=y_p(t)-y_m(t)\to 0 \tag{19}$$

To do this, we need to resolve an n-dimensional vector $x_p(t)$ from $\bar{r}$ number of time-varying linear equations represented by

$$\bar{C}_p x_p(t)=y_m(t) \tag{20}$$

This is difficult to achieve when the order of the output is high. However, in some applications, it is not necessary to match all outputs of the unknown system to the reference model and the matching condition will be relaxed. When a lower dimensional reference model output is used, we have $r\le\bar{r}$ for the $r\times\bar{r}$ matrix C_a and the rank condition

$$rank[C_a\bar{C}_p \vdots C_a\bar{C}_m]=rank[C_a\bar{C}_p] \tag{21}$$

is easier to satisfy. The time-varying vector $x_p(t)$ can be resolved to yield

$$C_a\bar{C}_p x_p(t)=C_m x_m(t) \tag{22}$$

or

$$y_p(t)=y_m(t) \tag{23}$$

An extreme case is to have C_a = [1 0_{r-1}] if $\bar{C}_{p11}\neq 0$, where 0_{r-1} is a *1* by *(r-1)* row vector of all zero entries. In this case, we always have $x_{p1}(t)=\left(\bar{C}_{m11}/\bar{C}_{p11}\right)x_{m1}(t)$ so that $y_p(t)=\bar{C}_{p11}x_p(t)=C_m x_m(t)=y_m(t)$. Let $C_p=C_a\bar{C}_p$ and define C_p^+ as pseudo-inverse matrix of C_p, that is $C_p^+C_p=I_n$, Eq. (5) becomes

$$e_{yp}(t)=C_m x_m(t)-C_p x_p(t) \tag{24}$$

If the matching condition is satisfied, we obtain an ideal state

$$x_p^*(t) = C_p^+ C_m x_m(t) \tag{25}$$

such that

$$y_p(t) = C_p x_p^*(t) = C_p C_p^+ C_m x_m(t) = y_m(t) \tag{26}$$

The existence of output matching is determined by matrices C_a, C_m, and $\overline{C}_p$. Although the values of C_a, and C_m are known, the ideal state $x_p^*(t)$ cannot be calculated since the value of $\overline{C}_p$ is unknown. We have shown that such $x_p^*(t)$ exists for many systems since the conditions stated in (21) are not restrictive. Next, we show that the vector $x_p(t)$ approaches the ideal state vector $x_p^*(t)$. That is, the state error vector

$$e_{xp}(t) = x_p^*(t) - x_p(t) \tag{27}$$

approaches zero with appropriate inputs, and the PID controller state error vector

$$e_{xf}(t) = e_{yf}(t) = y_f^*(t) - y_f(t) \tag{28}$$

vanishes while the controller output reaches the ideal value.

4. OUTPUT TRACKING CONDITIONS

The existence of the ideal state does not always yield a perfect tracking. This section derives the condition under which an ideal input exists to drive the system state to approach the ideal state asymptotically.

The derivative of (27) gives

$$\dot{e}_{xp}(t) = \dot{x}_p^*(t) - \dot{x}_p(t) \tag{29}$$

Inserting (1), (3), and (25) into (29), we have

$$\begin{aligned}
\dot{e}_{xp}(t) &= C_p^+ C_m \left(A_m x_m(t) + B_m u_m(t)\right) - \left(A_p x_p(t) + B_p u_p(t)\right) \\
&= C_p^+ C_m \left(A_m x_m(t) + B_m u_m(t)\right) - A_p x_p(t) \\
&\quad - B_p\left(K_{pe}(t) e_{yp}(t) - K_{pf}(t) y_f(t) + K_x(t) x_m(t) + K_u(t) u_m(t)\right) \\
&= \left(A_p - B_p K_{pe}(t) C_p\right) e_{xp}(t) + B_p K_{pf}(t) e_{xf} + Q(t)
\end{aligned} \tag{30}$$

where

$$\begin{aligned}
Q(t) &= B_p K_{pf}(t) y_f^*(t) + (C_p^+ C_m A_m - A_p C_p^+ C_m \\
&\quad - B_p K_x(t)) x_m(t) + (C_p^+ C_m B_m - B_p K_u(t)) u_m(t)
\end{aligned} \tag{31}$$

Defining the error dynamic equation of the PID controller as

$$\begin{aligned}
\dot{e}_{xf}(t) &= \dot{y}_f^*(t) - \dot{y}_f(t) \\
&= A_f y_f^*(t) - A_f y_f(t) - B_p C_p e_{xp}(t) \\
&= A_f e_{xf}(t) - B_p C_p e_{xp}(t),
\end{aligned} \tag{32}$$

we have the dynamic function for the feedback structure as below

$$\begin{bmatrix} \dot{e}_{xp}(t) \\ \dot{e}_{xf}(t) \end{bmatrix} = \begin{bmatrix} A_p - B_p K_{pe}(t) C_p & -B_p K_{pf}(t) \\ -B_f C_p & A_f \end{bmatrix} \begin{bmatrix} e_{xp}(t) \\ e_{xf}(t) \end{bmatrix} + \begin{bmatrix} Q(t) \\ 0 \end{bmatrix} \tag{33}$$

Since the unknown system is assumed to be controllable and observable, there exist constant matrices $K_{pe}(t) = \widetilde{K}_{pe}$, $K_{pf}(t) = \widetilde{K}_{pf}$, A_f, and B_f such that the meta-system is stable. Therefore, if $Q(t)=0$, $e_{xp}(t) \to 0$. A solution for $Q(t)=0$ is to assume that $y_f^*(t)$ is the linear combination of $x_m(t)$ and $u_m(t)$. That is $y_f^*(t) = S_x x_m(t) + S_u u_m(t)$, where the values of constant matrices S_x and S_u are introduced only for the convenience of theoretic discussion and are not needed in implementation. The condition for the existence of S_x and S_u are discussed as follows: If the meta-system is stable with $K_{pf}(t) = 0$ and

$$rank[B_p] = rank[B_p \,\vdots\, C_p^+ C_m A_m - A_p C_p^+ C_m \,\vdots\, C_p^+ C_m B_m], \tag{34}$$

there exist constant matrices $K_x(t) = \widetilde{K}_x$ and $K_u(t) = \widetilde{K}_u$ such that

$$\begin{aligned}
&\left(C_p^+ C_m A_m - A_p C_p^+ C_m - B_p \widetilde{K}_x\right) x_m(t) \\
&+ (C_p^+ C_m B_m - B_p \widetilde{K}_u) u_m(t) = 0
\end{aligned} \quad \text{or } Q(t) = 0. \tag{35}$$

In this case, the integrator does not contribute in the PID controller. Otherwise, the integrator is activated to balance and eliminate the term $Q(t)$. In general, if

$$\begin{aligned}
rank[B_p \widetilde{K}_{pf}] = rank[&B_p \widetilde{K}_{pf} \,\vdots\, C_p^+ C_m A_m - A_p C_p^+ C_m \\
&- B_p \widetilde{K}_x \,\vdots\, C_p^+ C_m B_m - B_p \widetilde{K}_u]
\end{aligned}, \tag{36}$$

there exist matrices $S_x = \widetilde{S}_x$, $S_u = \widetilde{S}_u$, $K_x(t) = \widetilde{K}_x$, and $K_u(t) = \widetilde{K}_u$ such that

$$B_p \widetilde{K}_{pf} \widetilde{S}_x = C_p^+ C_m A_m - A_p C_p^+ C_m - B_p \widetilde{K}_x \tag{37}$$

$$B_p \widetilde{K}_{pf} \widetilde{S}_u = C_p^+ C_m B_m - B_p \widetilde{K}_u \tag{38}$$

Therefore, $Q(t) = 0$.

Before discussing the adaptive control, it shows that if the system is known, we can satisfy the tracking conditions in (37) and (38) by manipulating matrices $\tilde{S}_x$, $\tilde{S}_u$, and $\tilde{K}_{pf}$. This is very important since it proves that the ideal inputs exist so that the tracking of the ideal states and desired trajectories is achievable. In practice, the system parameters are not known and the system is subject to input and output disturbances. An adaptive mechanism is used to adjust the matrices, $K_x(t)$, $K_u(t)$, and $K_{pf}(t)$ so that the state $x_p(t)$ of the not explicitly known system matches the ideal state $x_p^*(t)$ and the tracking output of the unknown system tracks the output of the reference model asymptotically. That is

$$y_p(t) \to C_p x_p^*(t) = C_m x_m(t) = y_m(t).$$

It is remarkable that $\tilde{K}_{pe}, \tilde{K}_{pf}, \tilde{K}_x$ and $\tilde{K}_u$ are virtual gain matrices used in proving the existence of output tracking. Neither their numerical values nor their implementation is required.

5. ADAPTIVE MECHANISM AND STABILITY

Since the system is not explicitly known, the feedback PID controller is needed to adaptively choose the values of K_P, K_I, and K_D (equivalently K_{pe}, K_{pf}, K_x, and K_u) in order to stabilize the dynamics in (33) and eliminate the unwanted terms in (35). The term asymptotic output reference model tracking means that the system output approaches the reference model output, $y_p(t) \to y_m(t)$, when the time is sufficiently large. When output tracking occurs, the corresponding state and control trajectories are defined to be the ideal state $x_p(t) = x_p^*(t) = C_p^+ C_m x_m(t)$ and ideal control command $u_p(t) = u_p^*(t)$ respectively. While

$$e_{yp}(t) = C_a\left(y_m(t) - y_p^*(t)\right) \to 0,$$

the system is driven by the ideal input $u_p^*(t)$. If the tracking conditions discussed in the last section are satisfactory, we show that the output tracking can be implemented adaptively.

To show the stability of the adaptive control mechanism, we introduce an ideal dynamic system without input and output disturbances as shown below

$$\dot{x}_P^*(t) = A_p x_P^*(t) + B_p u_p^*(t) \tag{39}$$

$$y_p^*(t) = C_p x_p^*(t) = y_m(t) \tag{40}$$

$$\dot{y}_f^*(t) = A_f y_f^*(t) \tag{41}$$

$$u_p^*(t) = -\tilde{K}_{pf} y_f^*(t) + \tilde{K}_x x_m(t) + \tilde{K}_u u_m(t) \tag{42}$$

Since the system is not known and the virtual matrices $\tilde{K}_{pe}, \tilde{K}_{pf}, \tilde{K}_x$ and $\tilde{K}_u$ cannot be computed. The adaptive structure is designed by computing the adaptive gain matrices $K_{pe}(t), K_{pf}(t), K_x(t)$ and $K_u(t)$ to eliminate the output tracking errors. Those adaptive gain matrices are used to replace the theoretical matrices $\tilde{K}_{pe}, \tilde{K}_{pf}, \tilde{K}_x$ and $\tilde{K}_u$ in order to stabilize the adaptive control system and to track the desired output. The adaptive gain matrices are defined as

$$K(t) = [K_e(t) \quad K_{px}(t) \quad K_{pu}(t)] \tag{43}$$

and

$$r(t) = \begin{bmatrix} e_{yv}(t) \\ x_m(t) \\ u_m(t) \end{bmatrix} \tag{44}$$

where $K_e(t) = [K_{pe}(t) \quad K_{pf}(t)]$ (45)

and $e_{yv}(t) = \begin{bmatrix} e_{yp}(t) \\ -y_f(t) \end{bmatrix}$, (46)

we rewrite Eq. (18) into a vector form

$$u_p(t) = K(t)r(t) \tag{47}$$

The adaptive gain matrix $K(t)$ is chosen to be a combination of proportional and integral (PI) terms as follows [1, 2]:

$$K(t) = K^P(t) + K^I(t) \tag{48}$$

where the proportional term is described by

$$K^P(t) = v(t)r^T(t)\bar{T} \tag{49}$$

and integral term is described by

$$K^I(t) = [v(t)r^T(t) - \sigma K^I(t)]T \tag{50}$$

with the initial gains given by

$$K^I(0) = [K_e^I(0) \quad K_{px}^I(0) \quad K_{pu}^I(0)] \tag{51}$$

The signal $v(t)$ is chosen based upon the Lyapunov stability analysis, which is in the form of

$$v(t) = Qe_{yv}(t) + GK(t)r(t) \tag{52}$$

Where

$$Q = [Q_p \quad Q_f] \tag{53}$$

The matrices T, $\bar{T}$, Q, and G are matrices selected by designers such that T and $\bar{T}$ are positive definite symmetric and positive semi definite symmetric, respectively, and such that Q and G satisfy the sufficient conditions for stability, and positive scalar σ is introduced to guarantee robustness in the presence of disturbances.

The stability of the adaptive system must be studied to insure all states and gains have bounded values. In order to simplify the stability proof, a meta-state model is used by defining meta-vectors:

$$x(t)=\begin{bmatrix} x_p(t) \\ x_f(t) \end{bmatrix} \text{ and } y(t)=\begin{bmatrix} y_p(t) \\ y_f(t) \end{bmatrix} \tag{54}$$

where $y_f(t)=x_f(t)$. Combining the state-variable equations of the unknown system, Eqs. (1) and (2) with the state-valuable equations of the adaptive PID controller in (47), to form a metastate valuable system described by

$$\dot{x}(t)=Ax(t)+BK(t)r(t)+d_i \tag{55}$$

$$y(t)=Cx(t)+d_o \tag{56}$$

where the system is described by meta-matrices:

$$A=\begin{bmatrix} A_p & 0 \\ -B_fC_p & A_f \end{bmatrix}, B=\begin{bmatrix} B_p \\ 0 \end{bmatrix}, C=\begin{bmatrix} C_p & 0 \\ 0 & I_r \end{bmatrix} \tag{57}$$

and the disturbances are described by metastates:

$$d_i(t)=\begin{bmatrix} E_p d_{ip}(t) \\ B_f(y_m(t)-d_{op}(t)) \end{bmatrix} \text{ and } d_o(t)=\begin{bmatrix} d_{op}(t) \\ 0 \end{bmatrix} \tag{58}$$

The adaptive control algorithm described in (55) and (56) is stable [15] if there exist a real symmetric positive definite matrix P and real matrix $\widetilde{K}_u$ and R, $R+R^T>0$, such that

$$P(A-B\widetilde{K}_eC)+(A-B\widetilde{K}_eC)^TP=-LL^T-R<0 \tag{59}$$

$$PB=C^TQ^T \tag{60}$$

where matrices T and $\overline{T}$ are positive definite symmetric and positive semi-definite symmetric, respectively. The sufficient conditions in (59) and (60) only assume that the metasystem *{A, B, C}* is ASPR. In this case, the unknown system $\{A_p, B_p, C_p\}$ is not directly restricted by ASPR conditions.

A less restrictive sufficient condition statement is developed based on BIBO stability analysis [9], which states that all states and errors in the adaptive system in (59) and (60) are bounded if there exist a real symmetric positive definite matrix P and real matrices L, W, $\widetilde{K}_u$, and R, $R+R^T>0$, such that

$$P(A-B\widetilde{K}_eC)+(A-B\widetilde{K}_eC)^TP=-LL^T-R<0 \tag{61}$$

$$PB=C^T(Q^T+\widetilde{K}_e^TG^T)-LW \tag{62}$$

$$W^TW=J+J^T \tag{63}$$

$$J+J^T+G+G^T<0 \tag{64}$$

The sufficient conditions in (61)-(64) do not restrict to the assumption that the unknown system $\{A_p, B_p, C_p\}$ is ASPR. Comparing (59) to (60), the later is much less restrictive due to the additional term LW.

6. EXAMPLE

A classic example studied by many authors [14, 15, 19, 23] is the so called Rohrs' example described by

$$\frac{y_p(s)}{u_p(s)}=\frac{2}{s+1}\frac{229}{s^2+30s+229} \tag{65}$$

The open loop system in (65) is stable but has a pair of complex unmodeled poles. The root locus of (65) shows that the dominant second order term of the corresponding closed loop system becomes unstable when the loop gain is larger than the admissible limit as shown in Fig. 3. This example is considered as a difficult case in adaptive control and is used to test various adaptive controllers.

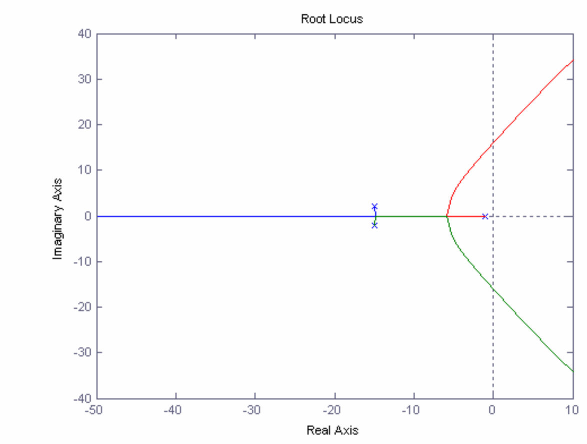

Fig. 3 Root locus plot of Rohrs example

The output of the system in (65) is required to follow the output of the reference model, which is shown below

$$\frac{y_m(s)}{u_m(s)}=\frac{1}{1+s/3} \tag{66}$$

To demonstrate the necessity of adopting an adaptive PID controller, the system response of using a fixed PI controller is tested by insert the function

$$H_0(s)=\frac{-(10s+35)}{s} \tag{67}$$

into the feedback loop. As shown in Fig. 4, the dominant second order term leads to a stable direction with the increasing of loop gain. The square wave response of the closed loop systems in (65) with the PI controller in (67) is stable but has a setting time greater than 500 seconds and a damping ratio for the dominant poles of less than 0.001 as shown in Fig. 5. The performance is unsatisfactory.

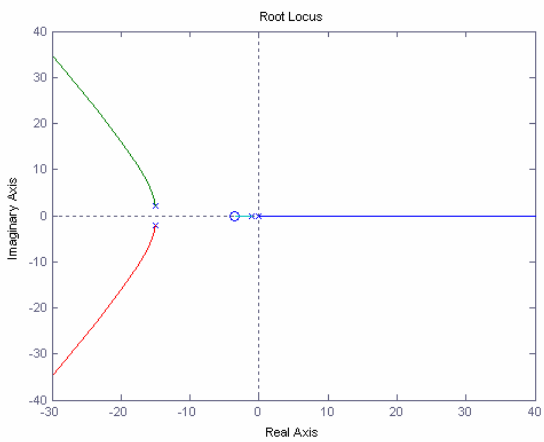

Fig. 4. Root locus plot of Rohrs example with PID

Since the mathematic description of the robot is not explicitly known and the parameters of the robot may vary unexpectedly, the choice of a proper PID controller becomes very difficult. Thus, an adaptive PID controller

$$H(s) = K_{pe} - \frac{K_{pf}}{s} \quad (68)$$

is inserted into the feedback loop. The state-variable description of (68) is

$$\dot{x}_f(t) = e_{yp}(t) \quad (69)$$

$$z_f(t) = -K_{pf}(t)x_f(t) + K_{pe}(t)e_{yp}(t) \quad (70)$$

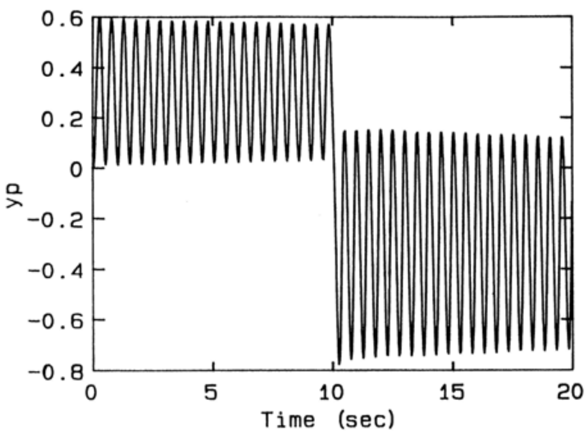

Fig. 5.. Output Tracking with PID Controller

The test is conducted by applying a square wave reference command u_m of magnitude 0.3 units and period of 20 seconds to the reference model in (66) to generate a desired trajectory y_m. The adaptive mechanism computes the parameters of the PI controller and the gains of the feedforward in order to stabilize the system and eliminate the output tracking errors. Q_p=57.14, Q_f=0, and G=0 are chosen in the simulation based on the design specifications [15]. As shown in Fig. 6, the stability of the closed loop system is significantly improved. The output of (65) tracks the output of the reference model in (66) asymptotically and achieves zero output tracking error in approximately two seconds.

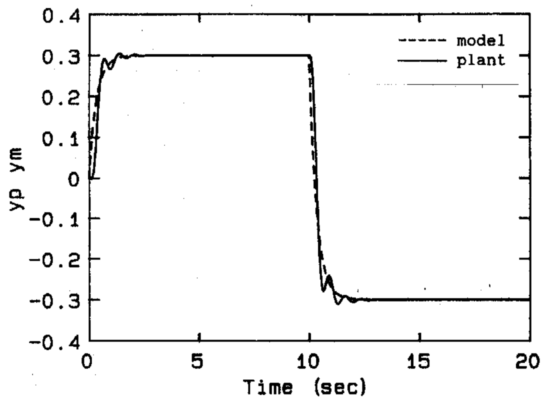

Fig. 6. Output tracking with Adaptive PID Controller

8. Conclusion

Adaptive PID controller based on DMROT is developed by using both feedforward and feed back adaptive mechanisms. The outputs of the not explicitly known MIMO system are forced to track the outputs of a known reference model asymptotically. This allows us to manipulate and control the multiple motions of a complex and not explicitly known robot using a single controller by simply operating on a known reference model. It also implies to manipulate an unstable and high-order robot by dealing with a stable and lower order reference model. The parameters of the PID controller are self-adjusted in time to achieve the best performance. The adaptive system tolerates the parameter change and input/output disturbances. Conditions for output matching and output tracking between the system and reference model are derived. A matching system outputs to reference model output (the ideal trajectory) exist if the rank condition in (21) is met. The system outputs track the reference model outputs if the rank condition in (34) or (36) is satisfied. The stability of the adaptive control is ensured for asymptotic or BIBO output tracking if the sufficient conditions in (61)-(64) are true. Simulation shows that the adaptive PID control eliminates output tracking error with a satisfactory performance.

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