Adaptive Output Following Control of a Networked Thermal Process

Mingcong DENG, Shengjun WEN, and Akira INOUE

Abstract—In this paper, an adaptive output following control based on command generator tracker (CGT) is proposed and applied to networked aluminum plate thermal process. One of the most important problems is the time delay in networked control system (NCS). An explorative technique is that the system with time delays is approximated as high order lag system. A fixed output feedback gain is discussed to guarantee robust stability of the system with delays. Meanwhile, the adaptive following control scheme is investigated to deal with the effect of network delay and uncertainties. Finally, networked control system for an aluminum plate thermal process is constructed and simulation and experimental results are given to show the effectiveness of the proposed design scheme compared with non-adaptive control approach.

Index Terms—Networked control system, thermal process, command generator tracker, robust parallel compensator, adaptive control

1. INTRODUCTION

In modern industrial control systems, sensors, actuators and controllers are usually located at the various place connected by communication network which gives rise to networked control systems. Such that network-based control emerged as a topic of significant interest in control field recently. One of the most important problems is the time delay in networked control system. It is well known that the occurrence of delay degrades the stability and performance of control systems [1, 2, 3]. For this challenging problem, one of the simplest and common techniques is to approximate the system with time delays as high order rational transfer function and to design model output following control based on CGT [8]. The CGT theory was first proposed for model following problem with known constant parameters in [4]. However, parameters of the approximating high order lag system are generally variable for time delays from network and system’s devices are time-varying and uncertainties. As a result, a weak tracking performance or even a weak oscillation may be caused for the networked control system. For this system with uncertainties, simple adaptive control based on CGT was employed by Deng, etc. in [5, 7], including feedback control to ensure robust stability and feedforward control to obtain effective tracking performance. But, it usually takes a long time to obtain reasonable adaptive feedback gain to satisfy the robust stability. For improving networked control system performance in this paper, a fixed output feedback gain is discussed to guarantee robust stability and an adaptive CGT algorithm is investigated to deal with the effect of network delays and uncertainties. That is, adaptive output following control scheme based on CGT is proposed for a networked aluminum plate thermal process.

The contents of this paper will be written as follows. In Section 2, a networked aluminum plate thermal process is introduced. And the model of the process is derived and problem statement is setup. After that, adaptive output following control based on CGT is presented. Simulation and experimental results show the effectiveness of the proposed method in Section 4. Finally, Section 5 draws the conclusion of this paper.

2. PROBLEM STATEMENT

2.1. Networked Aluminum Plate Thermal Process

A networked aluminum plate thermal process shown in Fig. 1 consists of two parts connected by LAN: (1) Computer PC1 as a controller; (2) Computer PC2, I/O board and aluminum plate setup as a controlled process [1].

![Fig. 1. A networked aluminum plate thermal system](image)

The experimental part is composed of a computer PC2 for delivering control command from PC1 to I/O board and delivering the process output information from I/O board to a digital I/O board, heater control circuit, thermometer circuit and an aluminum plate. The I/O board contains A/D, D/A and buffer boards. Heater control circuit contains a compensator, a counter oscillation circuit and solid-state relay. Computer PC1 demands to process a networked control using model output following control based on command generator tracker given in Section 3, consisting of the following two tasks: (1) Control the temperature of the aluminum plate; (2) Communication with PC2 to obtain the temperature data. The time delays are communication delays between PC1 and PC2 for delivering temperature data as $\tau_1$, and between PC1 and PC2 for sending control data to heater as $\tau_2$, where the delays $\tau_1$ and $\tau_2$ are measurable. Such that the total time delays...
defined as $T_D$ from getting the measured signal to sending the control signal to the process are equal to $\tau_1 + \tau_2$.

### 2.2. Modeling

The configuration of the process is shown in Fig. 2. Three laws are used to develop the mathematical model shown as follows.

1. Fourier’s law of heat conduction
   \[ q = -\lambda(d\theta/dn) \]  
   (1)

2. Newton’s law of cooling
   \[ q = \alpha(\theta - \theta_f) \]  
   (2)

3. Equation between heat capacity and objects and their specific heat
   \[ cmd\theta = dQ \]
   (3)

From Eqs. (1), (2) and (3), we can obtain a lumped parameter system, that is

\[
\frac{d(\theta - \theta_f)}{dt}mc = u - \alpha(\theta - \theta_f)(S_1 + 2S_3 - S_4) - 2\lambda S_2 \frac{\theta - \theta_f}{d_2}
\]

Define $y(t) = \theta - \theta_f$, then we take the Laplace transform of the above equation. Such that the transfer function of input-output can be derived as

\[
G(s) = \frac{Y(s)}{U(s)} = \frac{1}{mc\alpha S_1 + 2S_3 - S_4 + 2\lambda S_2 d_2}
\]

Consider the time delays caused by communication network, we obtain the model of the aluminum plate thermal process shown as follows

\[
G_p(s) = \frac{1}{729s + 9.656}e^{-T_D s}
\]

### 2.3. Problem Setup

We consider the time delays described by the following 1st, 2nd or higher order lag element such that

\[
e^{-T_D s} \approx \frac{1}{(1 + \frac{T_D}{n} s)^n}, n = 1, 2, \ldots
\]

The aluminum plate thermal process then can be rewritten as

\[
G_p(s) \approx G_p^*(s) = \frac{1}{(729s + 9.656)(1 + \frac{T_D}{n} s)^n}
\]

where $G_p^*(s)$ is a minimum phase process. That is, $G_p^*(s)$ is a approximate process with high order lag element. In fact, process parameters are variable and related to time delays $T_D$ for networked time delays are varying and uncertainties. The objective of this paper is to design a stable tracking controller for the approximate process by using adaptive output following control approach based on CGT.

### 3. Adaptive Output Following Control

The structure of adaptive output following control for the networked aluminum plate thermal process is described in Fig. 3.

From Eqs. (1), (2) and (3), we can obtain a lumped parameter system, that is

\[
\frac{d(\theta - \theta_f)}{dt}mc = u - \alpha(\theta - \theta_f)(S_1 + 2S_3 - S_4) - 2\lambda S_2 \frac{\theta - \theta_f}{d_2}
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**Fig. 2.** The configuration of the aluminum plate thermal process

Notations and parameters utilized are shown in Tables 1 and 2.

**TABLE I**

NOTATIONS FOR THE ALUMINUM PLATE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>Heat flux</td>
<td>W/m²</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Specific heat of aluminum</td>
<td>J/kgK</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Thermal conductivity</td>
<td>W/mK</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Heat transfer coefficient</td>
<td>J/m²K</td>
</tr>
<tr>
<td>$\theta_f$</td>
<td>Flowing air temperature</td>
<td>K</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Variation in heat capacities</td>
<td>J/kgK</td>
</tr>
</tbody>
</table>

**TABLE II**

PARAMETERS FOR THE CONFIGURATION OF THE ALUMINUM PLATE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>900 J/kgK</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>25 w/m²K</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>200 w/mK</td>
</tr>
<tr>
<td>$d$</td>
<td>2700 kg/m³</td>
</tr>
<tr>
<td>$m$</td>
<td>$d_1 \times h \times l \times d_1$ kg/m³</td>
</tr>
</tbody>
</table>

**Fig. 3.** Model output following control

where $u^*(t)$ is the adaptive feedforward input based on reference model and error $e_{ya}(t)$ between reference model output $y_m(t)$ and the augmented process output $y_a(t)$, and $u(t)$ is feedback control input to ensure closed-loop stability of the
system. The main purpose is that the augmented process output follows reference model output.

3.1. Robust parallel compensator

Model output following control does not require the reference model to be of the same order as the process and the knowledge of the process order is not needed. However, instead of the order requirement, other conditions need to be satisfied. That is, the process $G_p(s)$ is required to be ASPR [2]. Here, $G_p(s)$ is non-ASPR process in the networked aluminum plate thermal system for the relative degree $n + m_2 - m_1 \geq 2$ ($m_2$ and $m_1$ are denominator and numerator order of $G(s)$).

Such that robust parallel compensator (RPC) $F(s)$ is designed to transform the non-ASPR process $G_p(s)$ into an ASPRness. For the approximated process $G_p(s)$ with high order lag element, we design a robust parallel compensator $F(s)$ that makes the resulting augmented process $\hat{G}(s) = G_p(s) + F(s)$ satisfying ASPR condition as follows:

$$F(s) = \frac{s}{(s + \frac{n}{T_D})(729s + 9.656)} \sum_{i=1}^{n} F_{1i}(s)$$

$$+ \sum_{j=1}^{m_2-m_1-1} \delta^j F_{2j}(s)$$

(9)

$$F_{1i}(s) = \left(\frac{n}{T_D} \right) i^{-1}$$

$$F_{2j}(s) = \frac{\beta_1 n_j(s)}{d_j(s)}$$

(10)

where $\delta$ is small positive constant, $d_j(s)$ is monic stable polynomial of any order $n_d_j \geq m_2 - m_1 - j$ and $n_j(s)$ is monic stable polynomial of any order $m_{n_j} = n_d_j - (m_2 - m_1 - j)$. The parameter $\beta$ is chosen such that the following polynomial is the Hurwitz polynomial.

$$r(s) = \beta_{m_2-m_1-1}s^{m_2-m_1-1} + \cdots + \beta_1 s + \beta_0$$

(11)

Then, the augmented process $\hat{G}(s)$ can be re-expressed as

$$\hat{G}(s) = \frac{1}{729s + 9.656} + \sum_{j=1}^{m_2-m_1-1} \delta^j F_{2j}(s)$$

(12)

where, $\hat{G}(s)$ is considered to be consisted of a process $1/(729s + 9.656)$ and a small RPC $\sum_{j=1}^{m_2-m_1-1} \delta^j F_{2j}(s)$. So, it can be concluded that the augmented process $\hat{G}(s)$ with the RPC $F(s)$ can be made ASPR by choosing a sufficiently small $\delta$. That is, we can transform the approximated process $G_p^*(s)$ into an ASPR augmented process $\hat{G}(s)$ using a RPC.

3.2. Robust stable feedback control

Next, we consider the augmented process $G_a(s)$ for the actual process $G_p(s)$ shown as follows

$$G_a(s) = G_p(s) + F(s)$$

(13)

Denoting the model-process mismatch by

$$\tilde{G}(s) = G_p(s) - G_p^*(s)$$

as the additive uncertainty, we have

$$G_a(s) = \tilde{G}(s) + \hat{G}(s)$$

(14)

So, for the augmented process $G_a(s)$, we design the closed-loop system with stable output feedback controller $C(s) = k_e$ which is described as a simple configuration with $\tilde{G}(s)$ in forward path and $M(s)$ in the feedback path shown as Fig. 4.

![Fig. 4. The configuration of the output feedback control system](image)

Then, $M(s)$ can be derived as

$$M(s) = \frac{-k_e}{1 + k_e \hat{G}(s)}$$

(15)

For the closed-loop system with feedback gain $k_e$, if

$$\|\hat{G}(j\omega)M(j\omega)\| < 1, \forall \omega \in [0, \infty)$$

(16)

then the robust stability can be guaranteed. Such that, from Eq. (13), (17) and (13), a sufficient condition for the closed-loop stability is given as

$$\|\hat{G}(j\omega)\| < \|\frac{1}{k_e} + \frac{1}{j\omega + \frac{9}{729} + 9.656} + \sum_{i=1}^{m_2-m_1-1} \delta^i F_{2i}(j\omega)\|$$

(17)

Therefore, the augmented process $G_a(s)$ for the actual process $G_p(s)$ with the time delays given in (6) can be made robust stable by feedback control, where feedback gain $k_e$ is irrespective of the order of the lag element. That is, we can select any order for the lag element to ensure the robust stability of the control system. Meanwhile, the time delays and uncertainties are bound such that a boundary of the feedback gain $k_e$ can be derived according to time delays and uncertainties. Next, adaptive following control scheme based on CGT will be presented.

3.3. Adaptive following control scheme

In Fig. 3, $G_p(s)$ is the controlled process and $y_p(t)$ and $u_p(t)$ are the process output and the control signal respectively. Let a minimal realization of approximated process $G_p^*$ be $(A_p, b_p, e_p)$ which is controllable and observable shown as follows.

$$\dot{x_p}(t) = A_p x_p(t) + b_p u_p(t)$$

$$y_p(t) = e_p^T x_p(t)$$

(18)
The stable reference model is proposed by following state-space representation:

\[
\dot{x}_m(t) = A_m x_m(t) + b_m u_m(t) \\
y_m(t) = c^T_m x_m(t)
\]  

where \( u_m(t) \) and \( y_m(t) \) are the reference model input and output, respectively. For the augmented process \( \hat{G}(s)=(G_p^*(s)+F(s)) \) and the reference model, we impose the following assumptions:

1. For the ASPR augmented process \( \hat{G}(s) \) and the feedback gain \( k_e \), the inequality (19) is satisfied.
2. The reference input \( u_m(t) \) and its higher derivatives \( u^{(i)}_m(t) (i=1, \ldots, n+m_2-m_1) \) are bounded.

The objective is to make the augmented process output \( y_a(t) \) track the reference model output \( y_m(t) \) such that the model output adaptive following control based on the CGT theory is proposed. Under these assumptions, we design the control input as follows:

\[
u_p(t) = u(t) + u^*(t)
\]

where \( u(t) \) is the feedback input which guarantees the stability given by

\[
u(t) = -k_e e_{ya}(t)
\]

\[
e_{ya}(t) = y_m(t) - y_a(t)
\]

\[
y_a(t) = y_p(t) + y_f(t)
\]

where \( y_f(t) \) is output of the RPC which is presented by the following state-space representation.

\[
\dot{x}_f(t) = A_f x_f(t) + b_f u(t) \\
y_f(t) = c^T x_f(t)
\]

\( u^*(t) \) is the adaptive feedforward input based on the CGT theorem given by

\[
u^*(t) = k(t)^T z(t)
\]

where

\[
k(t)^T = [k_{xm}(t)^T, k_{um}(t)^T] \\
z(t)^T = [x_m(t)^T, u_m(t)^T]
\]

Adaptive rule is shown as follows:

\[
k(t) = k_1(t)^T + k_p(t)^T
\]

where

\[
\dot{k}_1(t)^T = \Gamma_I z(t) e_y(t) - \delta(t) k_1(t) \\
\dot{k}_p(t)^T = \Gamma_p z(t) e_y(t) \\
\delta(t) = \frac{\delta_1 e_y(t)^2 + \delta_2}{1 + e_y(t)^2}, \delta_1 > 0, \delta_2 > 0, \\
\Gamma_I = \Gamma_I^T > 0, \Gamma_p = \Gamma_p^T > 0
\]

where \( \delta_1, \delta_2, \Gamma_I \) and \( \Gamma_p \) are adaptive control parameters. The stability of the adaptive control system can be ensured under the mentioned assumptions. For the proof see reference [6].

4. SIMULATION AND EXPERIMENTAL RESULTS

In this section, the simulation and experiment are utilized to show the efficiency of the proposed design scheme compared with non-adaptive control method for the aluminum plate thermal process. The communication time delays and process devices’ delays in experimental environment are measured firstly shown as Fig. 5. The time delays consist of delivering the temperature signal from the sensors to the controller and sending the control signal from the controller to the process. As a result, it is shown that the delays are varying about 6 ~ 7 sec in this communication environment.

\[
G_p^*(s) = \frac{1}{(729s + 9.656)(1 + \frac{s}{n}s)^n}
\]

Let \( n = 3 \) to approximate the time delays, we have

\[
G_p^*(s) = \frac{1}{729s + 9.656} \cdot \frac{1}{(1 + \frac{s}{n}s)^n}
\]

We obtain the minimal realization of \( G_p^*(A_p, b_p, c_p) \) given as

\[
A_p = \begin{bmatrix}
-1.2990 & -0.5681 & -0.3441 & -0.0667 \\
1.0000 & 0 & 0 & 0 \\
0 & 0.2500 & 0 & 0 \\
0 & 0 & 0.0625 & 0
\end{bmatrix}
\]

\[
B_p = \begin{bmatrix}
0.0625 \\
0 \\
0
\end{bmatrix}
\]

\[
C_p = \begin{bmatrix}
0 & 0 & 0 & 0.1106
\end{bmatrix}
\]
The reference model is given as follows

\[
G_m(s) = \frac{1}{60s + 1}
\]

\[
A_m = -0.0167
\]

\[
B_m = 0.1250
\]

\[
C_m = 0.1333 \quad (30)
\]

and \( F(s) \) is given as

\[
F(s) = \frac{s^4 + 1.714s^3 + 1.102s^2 + 0.2362s + 0.3258}{729s^5 + 1259s^4 + 1259s^3 + 240.2s^2 + 27.63s + 0.3258} \quad (31)
\]

We select the output feedback gain as \( k_e = 100 \) to ensure the robust stability of the controlled system. Such that \( u(t) \) can be calculated as follows

\[
u(t) = -100e_y(t) - u^*(t) \quad (32)
\]

The robust stable margin of the controlled system is illustrated in Fig. 6.

![Fig. 6. The robust margin of the controlled system](image)

Adaptive control parameters are designed as follows:

\[
\Gamma_I = \text{diag}[0.001, 0.02]
\]

\[
\Gamma_P = \text{diag}[0.5, 0.5]
\]

\[
\delta_1 = 0.0001
\]

\[
\delta_2 = 0.0002
\]

Non-adaptive control feedforward input compared is given as

\[
u^*(t) = -0.2952x_m(t) + 11.8703u_m(t) \quad (33)
\]

4.1. Simulation Result

Time delay \( T_D \) is equal to 7[s] for simulation. Fig. 7 and Fig. 8 show the simulation results of non-adaptive and adaptive control scheme when \( u_m(t) = 1(t > 0) \). These show that the process output can track effectively the constant reference model output for two approaches.

![Fig. 7. The non-adaptive simulation result of constant reference input](image)

![Fig. 8. The adaptive simulation result of constant reference input](image)

When the input of the reference model is variable given as follows

\[
u_m = \begin{cases} 
1 & 0 \leq t < 500 \\
2 & 750 \leq t < 1250 \\
0 & 500 \leq t < 750, 1250 \leq t 
\end{cases} \quad (34)
\]

the process output responses are shown in Fig. 9 and Fig. 10 respectively. When the process is heated, the attentive tracking results have been obtained. But, there is a smaller delay for adaptive control method during cooling. It shows that under variable reference input there is better tracking effect for adaptive control.

4.2. Experimental Result

Fig. 11 and Fig. 12 show the experimental result of the constant reference input. Fig. 13 and Fig. 14 are the experimental result of the variable reference input.
From Fig. 11, Fig. 12, Fig. 13 and Fig. 14, when the reference inputs are changed, we can see that there are a little delays during warming up and explicit delays during cooling down for natural cooling. In Fig. 11 and Fig. 13, although good tracking results are obtained, there exist small oscillations for the augmented process output. To resolve this problem, adaptive CGT algorithm is employed for the above process, where CGT condition is assumed to be satisfied. As a result, small oscillations are cancelled and a desired control performance is obtained in Fig. 12 and Fig. 14.

The above simulation and experimental results show the effectiveness of adaptive output following control scheme based on CGT. Compared with non-adaptive control method, it is better to resolve the problem of time delays, and more effective to realize tracking control.
5. CONCLUSIONS

In this paper, adaptive output following control is investigated and applied to a networked aluminum plate thermal process. The modeling and problem statement are given after analyzing networked aluminum plate thermal process system structure. An explorative technique is utilized to approximate time delays as high order system with lag element. Then, a RPC is designed to transform the augmented process with high order system into ASPR. After that, a fixed output feedback gain is obtained to guarantee robust stability of the system and an adaptive output following CGT algorithm is presented to cancel the effect of networked time delays and uncertainties. Finally, simulation and experimental results are given to show the effectiveness of the proposed method compared with non-adaptive control approach.

REFERENCES


